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CONTRIBUTION OF HIGHER-ORDER DISPERSION TO NONLINEAR ELECTRON-ACOUSTIC SOLITARY WAVES IN A RELATIVISTIC ELECTRON BEAM PLASMA SYSTEM

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Abstract

The nonlinear properties of solitary wave structures are reported in an unmagnetized collisionless plasma comprising of cold relativistic electron fluid, Maxwellian hot electrons, relativistic electron beam, and stationary ions. The Korteweg--de Vries (KdV) equation has been derived using a reductive perturbation theory. As the wave amplitude increases, the width and velocity of the soliton deviate from the prediction of the KdV equation i.e. the breakdown of the KdV approximation. On the other hand, to overcome this weakness we extend our analysis to obtain the KdV equation with fifth-order dispersion term. The solution of the resulting equation has been obtained.

Keywords: *Electron acoustic waves ; Relativistic electrons; KdV equation; Reductive perturbation ; Higher-order dispersion; Solitary solution*

INTRODUCTION

Plasma physics has a rich variety of nonlinear structure and modes. For instance, electron-acoustic waves (EAWs) are high frequency (in comparison with the ion plasma frequency) electrostatic modes. EAWs may exist in plasmas consisting of positive ions and two distinct electron species, one cool and one hot, at high frequencies (but below the plasma frequency) such that the ion dynamics plays no essential role [1-2]. In such plasmas, the cooler of the two electron components provides the inertial effects needed to sustain the wave, as the ions do in the usual description of ion-acoustic waves. Also its propagation has been invoked to explain the emissions at frequencies between the ion and the electron plasma frequencies in the cusp of terrestrial magnetosphere, Earth bow shock, and the heliospheric termination shock [3-8]. Solitary electron acoustic structures has been studied by many authors, e.g., by Refs. [5,9] and [10]. In [11] Bharuthram studied the theory of electron-acoustic instability driven by field-aligned hot electron beam in a three-component plasma. Berthomier et al.[12] investigated the electron-acoustic solitons in an electron-beam plasma system with isothermal hot electrons. It was found that, when the electron beam taken into account in such plasma model allows the existence of new EA solitons with velocity related to the beam velocity. In addition, the second electron population modifies the topology of the roots of the linear dispersion relation in the phase velocity space. A generalized three-component model suggested before in [11] have been studied by Singh and Lakhina[13], including the hot electron component in addition to warm electron beam. Tagare et al[14] examined the generation of small amplitude electron-acoustic solitons in a magnetized plasma with four components namely, cold electron beam and background plasma electrons and two temperature ion plasma. In [15], Sahu and Roychoudhurya derived the exact Sagdeev pseudopotential for electron-acoustic waves in a two electron temperature plasma (cold and hot electrons) in the presence of relativistic electron beam plasma using a vortex-like distribution for trapped electrons. It was found that the relativistic effect restricts the region of existence for solitary waves. Investigations of small-amplitude electron-acoustic waves (EAWs) usually describe the evolution of the wave by Korteweg--de Vries (KdV) equation. The KdV equation is derived from the basic set of equations for plasma by, for example, the reductive perturbation theory [16]. The resulting equation contains the lowest-order nonlinearity and dispersion, and consequently can describe a wave of only small amplitude. As the wave amplitude increases, the width and velocity of a soliton deviate from the prediction of the KdV equation: the breakdown of the KdV approximation. To describe the (EAWs) of larger amplitude, the higher-order nonlinear and depressive effects have to be taken into account [17-21]. To the author's knowledge, No attention has been paid to investigate the effect of higher-order solutions in presence of relativistic electron beam to the EAWs in plasmas. So, the main concern of this work is to study of both the relativistic electrons and higher-order dispersive corrections on the amplitude and width of (EA) solitary waves in an unmagnetized collisionless plasma consists of a cold relativistic electron fluid, Maxwellian hot electrons, a relativistic electron beam and stationary ions. This paper is organized as follows; in Section 2, we present the basic set of fluid equations governing our plasma model. In Section 3, the nonlinear EAWs are investigated through the derivation of a Korteweg-de Vries (KdV) equation that contains the lowest-order nonlinearity and dispersion. In Section 4, the KdV equation with the fifth-order dispersion term is employed and its higher-order solution are obtained. Finally, some conclusions and discussions are given in Section 5.

BASIC EQUATIONS FOR RELATIVISTIC ELECTRON BEAM PLASMA SYSTEM

We consider a homogeneous system of an unmagnetized collisionless plasma consisting of a cold relativistic electron fluid, Maxwellian hot electrons , relativistic electron beam and stationary ions. For one dimensional propagation of a small- but finite amplitude nonlinear EAWs, the dynamics of the relativistic electron fluid can be written as:

$$\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0 \tag{1.a}$$

$$\frac{\partial(\gamma_c u_c)}{\partial t} + u_c \frac{\partial(\gamma_c u_c)}{\partial x} - \alpha \frac{\partial \phi}{\partial x} + 3\frac{\alpha}{\theta}(1+\alpha+\beta)^2 n_c \frac{\partial n_e}{\partial x} = 0, \tag{1.b}$$

$$\frac{\partial n_b}{\partial t} + \frac{\partial (n_b u_b)}{\partial x} = 0, \tag{1.c}$$

$$\frac{\partial(\gamma_b u_b)}{\partial t} + u_b \frac{\partial(\gamma_b u_b)}{\partial x} - \alpha \frac{\partial \phi}{\partial x} + 3\frac{\alpha}{\sigma}\frac{(1+\alpha+\beta)^2}{\beta^2} n_b \frac{\partial n_b}{\partial x} = 0, \tag{1.d}$$

$$\frac{\alpha}{(1+\alpha+\beta)}\frac{\partial^2\phi}{\partial x^2}-n_c-n_h-n_b+1=0, \tag{1.e}$$

$$\frac{(1+\alpha+\beta)}{\alpha}n_h-\exp(\phi)=0\,. \tag{1.f}$$

where

$$\gamma_c=(1+\frac{u^2{}_c}{2c^2}),\gamma_b=(1+\frac{u^2{}_b}{2c^2}).$$

In the above equations, $n_{c,h,b}$ are the densities of the three electron population $u_{c,b}$ are velocities of the cold and beam electrons, respectively, and ϕ is the electrostatic potential. In Eqs. (1) these quantities have been normalized to the total unperturbed density $n_0=n_{c0}+n_{h0}+n_{b0}$, the electron acoustic velocity $u_{ea}=(n_{c0}/n_{h0})^{1/2}u_{Th}$, where $u_{Th}=(K_BT_h/m_e)^{1/2}$ is the hot electron thermal velocity and to $K_BT_h/e,$ respectively. Time t and space variables x are normalized, respectively, to the cold electron plasma period $\omega\ {}_{pc}{}^{-1}=(m_e/4\pi n_{c0}e^2)^{1/2}$ and the hot electron Debye length $\lambda_{Dh}=(K_BT_h/4\pi n_{h0}e^2)^{1/2},$ where K_B is Boltzmann's constant and m_r is the mass of electron.

We have introduced the following quantities, which will be used in our parametric study:

$$\alpha=\frac{n_{h0}}{n_{c0}},\beta=\frac{n_{b0}}{n_{c0}}, \tag{2}$$

$$\theta=\frac{T_h}{T_c},\sigma=\frac{T_h}{T_b}. \tag{3}$$

NON LINEAR WAVES

According to the general method of reductive perturbation theory, we introduce the slow stretched co-ordinates:

$$\tau=\varepsilon^{3/2}t,\ \ \xi=\varepsilon^{1/2}(x-\lambda t)\,, \tag{4}$$

where ε, is a small dimensionless expansion parameter and λ is the wave speed normalized by u_{ea}. All physical quantities appearing in (1) are expanded as a power series in ε about their equilibrium values as:

$$\left.\begin{aligned} n_c&=1-\delta_1+\varepsilon n_{c1}+\varepsilon^2n_{c2}+\varepsilon^3n_{c3}+.............,\\ u_c&=u_{c0}+\varepsilon u_{c1}+\varepsilon^2u_{c2}+\varepsilon^3u_{c3}+.................,\\ n_b&=\delta_1+\varepsilon n_{b1}+\varepsilon^2n_{b2}+\varepsilon^3n_{b3}+...................,\\ u_b&=u_{b0}+\varepsilon u_{b1}+\varepsilon^2u_{b2}+\varepsilon^3u_{b3}+.................,\\ \phi&=\varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+..................,\end{aligned}\right\} \tag{5.a}$$

We impose the boundary conditions that as

$$|\xi|\to\infty,n_c=1-\delta_1,n_b=\delta_1,u_c=u_{c0},u_b=u_{b0},\phi=0. \tag{5.b}$$

Substituting (4) and (5) into the system of equations (1), and equating coefficients of like powers of ε . From the lowest-order equations in ε, the following results are obtained :

$$n_{c1} = b_1\phi_1, u_{c1} = b_2\phi_1, n_{b1} = b_3\phi_1, u_{b1} = b_4\phi_1 \tag{6}$$

where

$$b_1 = \frac{\alpha(-1+\delta_1)}{-a_1 + a_1\delta_1 + \rho^2\gamma_1}, b_2 = \frac{\alpha\rho}{-a_1 + a_1\delta_1 + \rho^2\gamma_1},$$

$$b_3 = \frac{-\alpha\delta_1}{(\rho + u_{c0} - u_{b0})^2\gamma_{11} - a_2\ \delta_1}, b_4 = \frac{\alpha(\rho + u_{c0} - u_{b0})}{-((\rho + u_{c0} - u_{b0})^2\gamma_{11}) + a_2\ \delta_1}.$$

with

$$\rho = \lambda - u_{c0}, \gamma_1 = 1 + \frac{3u_0^{\ 2}}{2c^2}, \gamma_{11} = 1 + \frac{u_{b0}^{\ 2}}{2c^2},$$

$$a_1 = \frac{-3\alpha(1+\alpha+\beta)^2(-1+\delta_1)}{\theta}, a_2 = \frac{3\alpha(1+\alpha+\beta)^2\delta_1}{\beta^2\sigma}.$$

Poisson's equation gives the linear dispersion relation

$$\frac{-\alpha}{1+\alpha+\beta} - b_1 - b_3 = 0. \tag{7}$$

If we consider the coefficients of o(ε^2), we obtain with the aid of equations (7) the following set of equations:

$$b_1\frac{\partial\phi_1}{\partial\tau} - \rho\frac{\partial n_{c2}}{\partial\xi} + (1-\delta_1)\frac{\partial n_{c2}}{\partial\xi} + 2b_1b_2\phi_1\frac{\partial\phi_1}{\partial\xi} = 0, \tag{8}$$

$$-n_{c2} - n_{b2} - \frac{\alpha}{2(1+\alpha+\beta)}\phi_1^{\ 2} - \frac{\alpha}{(1+\alpha+\beta)}\phi_2 + \frac{\alpha}{(1+\alpha+\beta)}\frac{\partial^2\phi_1}{\partial\xi^2} = 0, \tag{9}$$

$$b_2\gamma_1\frac{\partial\phi_1}{\partial\tau} + a_1\frac{\partial n_{c2}}{\partial\xi} - \rho\gamma_1\frac{\partial n_{c2}}{\partial\xi} + (a_3b_1^{\ 2} + b_2^{\ 2}(\gamma_1 - 2\rho\gamma_2))\phi_1\frac{\partial\phi_1}{\partial\xi} - \alpha\frac{\partial\phi_2}{\partial\xi} = 0\ , \tag{10}$$

$$b_3\frac{\partial\phi_1}{\partial\tau} - (\rho + u_{c0} - u_{b0})\frac{\partial n_{b2}}{\partial\xi} + \delta_1\frac{\partial n_{b2}}{\partial\xi} + 2b_3b_4\phi_1\frac{\partial\phi_1}{\partial\xi} = 0, \tag{11}$$

$$b_4\gamma_{11}\frac{\partial\phi_1}{\partial\tau} + a_2\frac{\partial n_{b2}}{\partial\xi} + b_6\frac{\partial n_{b2}}{\partial\xi} + (a_4b_3^{\ 2} + b_4^{\ 2}b_5)\phi_1\frac{\partial\phi_1}{\partial\xi} - \alpha\frac{\partial\phi_2}{\partial\xi} = 0. \tag{12}$$

Eliminate the second order perturbed quantities n_{c2}, n_{b2}, u_{c2} , $u_{b2}$and Φ_2 from equations (8-12), we obtain the following KdV equation for the first-order perturbed potential:

$$\frac{\partial\ \phi_1}{\partial\tau} + A\phi_1\frac{\partial\ \phi_1}{\partial\xi} + \frac{B}{2}\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \tag{13}$$

where,

$$A = \frac{A_2}{A_1}, B = \frac{A_3}{A_1},$$

$$A_1 = -(b_3 b_6 (\rho^2 \gamma_1 + a_1(-1+\delta_1)) - b_4 \gamma_{11} (\rho^2 \gamma_1 + a_1(-1+\delta_1))\delta_1 - \rho b_1 \gamma_1 (-b_6 (\rho + u_{c0} - u_{b0}))$$
$$- a_2 \kappa \delta_1) + b_2 \gamma_1 (-1+\delta_1)(-(b_6 (\rho + u_{c0} - u_{b0})) - a_2 \delta_1)),$$

$$A_2 = (2\rho b_1 b_2 \gamma_1 - \frac{\alpha(-(\rho^2 \gamma_1) - a_1(-1+\delta_1))}{1+\alpha+\beta} - a_3 b_1^{\,2}(-1+\delta_1) - b_2^{\,2}(\gamma_1 - 2\rho\gamma_2)(-1+\delta_1))(-(b_6$$
$$(\rho + u_{c0} - u_{b0})) - a_2\delta_1) - (\rho^2\gamma_1 + a_1(-1+\delta_1))(-2b_3 b_4 b_6 + a_4 b_3^{\,2}\delta_1 + b_4^{\,2} b_5 \delta_1),$$

$$A_3 = \frac{\alpha(\rho^2\gamma_1 + a_1(-1+\delta_1))(b_6(\rho + u_{c0} - u_{b0}) + a_2\delta_1)}{1+\alpha+\beta}.$$

Let us introduce the variable,

$$\eta = \xi - .\vartheta\tau \tag{14}$$

where η is the transformed coordinates and $.\vartheta$ is the arbitrary parameter similar to Mach number which allows the possibility of solitons moving with different velocity than the phase velocity of the wave. Integrating Eq. (13) with respect to the variable η and using the boundary conditions(5.b), we obtain the solution:

$$\phi_1 = \phi_0 \sec h^2 (D\eta) \tag{15}$$

where the peak soliton amplitude ϕ_0 and the soliton width D^{-1} are given by

$$\phi_0 = \frac{3.\vartheta}{A}, D^{-1} = \sqrt{\frac{2B}{.\vartheta}}.$$

HIGHER ORDER CORRECTION

It is clear that Eq. (13) contains the lowest-order nonlinearity and dispersion, and consequently can describe a wave of only small amplitude. As the wave amplitude increases, the width and velocity of a soliton deviate from the prediction of the KdV equation (13), i.e., the breakdown of the KdV approximation. So, to describe the EAWs of large amplitude, the higher-order nonlinearity and dispersive effect have to taken into account. For this end, the higher-order approximation of the reductive perturbation method has been known to be a powerful tool. A method of perturbation is employed to solve the following higher-order dispersion equation of the type:

$$\frac{\partial\psi}{\partial\tau} + A\psi\frac{\partial\psi}{\partial\xi} + \frac{B}{2}\frac{\partial^3\psi}{\partial\xi^3} + \varepsilon\frac{\partial^5\psi}{\partial\xi^5} = 0, \tag{16}$$

where ε is a smallness parameter. If ε is zero, last equation is reduced to the well-known KdV equation (13). The last term of the left-hand side in Eq. (16) is a perturbation added to the KdV equation (13) as a higher-order dispersive effect. Because of this term, we cannot solve the above equation exactly and have to rely on a perturbation method. However, the higher-order equation includes secular terms. The elimination of these secular terms will modify the soliton velocity, i.e., the secularity in the higher- order is renormalized to the velocity.

To solve Eq. (16), substituting (14) into (16), we get

$$-\vartheta\frac{\partial\psi}{\partial\eta}+A\phi_1\frac{\partial\psi}{\partial\eta}+\frac{B}{2}\frac{d^3\phi_1}{d\eta^3}+\varepsilon\frac{d^5\psi}{d\eta^5}=0, \tag{17}$$

To solve Eq. (17), we have presented a simple method, Watanabe and Jiang[22], for obtaining higher-order solitary wave solutions. According to this method, we expand ψ(η) and ϑ with respect to the smallness parameter ε as ,

$$\begin{aligned}\psi&=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\varepsilon^3\psi_3+\ldots\ldots,\\ \vartheta&=\vartheta_0+\varepsilon\vartheta_1+\varepsilon^2\vartheta_2+\varepsilon^3\vartheta_3+\ldots\ldots\ldots,\end{aligned} \tag{18}$$

Substituting (18) into (17) and collecting together the coefficient of like power in ε one obtains the following equations:

$$\varepsilon^0:-\vartheta_0\frac{d\psi_0}{d\eta}+A\psi_0\frac{d\psi_0}{d\eta}+\frac{B}{2}\frac{d^3\psi_0}{d\eta^3}=0, \tag{19.a}$$

$$\varepsilon^1:L\phi_1=\vartheta_1\frac{d\psi_0}{d\eta}-\frac{d^5\psi_0}{d\eta^5}, \tag{19.b}$$

$$\varepsilon^2:L\phi_2=\vartheta_1\frac{d\psi_1}{d\eta}+\vartheta_2\frac{d\psi_0}{d\eta}-A\psi_1\frac{d\psi_1}{d\eta}-\frac{d^5\psi_1}{d\eta^5}, \tag{19.c}$$

$$\varepsilon^3:L\phi_3=\vartheta_3\frac{d\psi_0}{d\eta}+\vartheta_2\frac{d\psi_1}{d\eta}+\vartheta_1\frac{d\psi_2}{d\eta}-A\,\frac{d(\psi_1\psi_2)}{d\eta}-\frac{d^5\psi_2}{d\eta^5}. \tag{19.d}$$

where the operator L is represented by

$$L=-\vartheta_0\frac{d}{d\eta}+A\frac{d}{d\eta}\psi_0+\frac{B}{2}\,\frac{d^3}{d\eta^3}. \tag{20}$$

This operator is the one-variable version of the linearized KdV operator.

Equations(19), include the third-order, and the higher-order derivatives have a secular terms as the higher-order equations of the reductive perturbation method, and the elimination of secular terms determines the velocity correction. We solve Equations. (19) successively under boundary conditions that ψ_i, $\frac{d\,\psi_i}{d\,\xi}$, and $\frac{d^2\psi_i}{d\xi^2}$ (i = 0,1,2,.....) vanish at η=±∞ . Equation. (19.a) yields a solitary wave solution in the form

$$\psi_0=\psi_m\sec h^2(D\eta), \tag{21}$$

where the soliton amplitude ψ_m and the soliton width D^{-1} are given by

$$\psi_m=\frac{3\vartheta_0}{A},D^{-1}=\sqrt{\frac{2B}{\vartheta_0}}.$$

The solution given by equation (21) is just one soliton solution of KdV equation.

In the next order of ε, substituting equation (21) into (19.b), we arrive at

$$\begin{aligned}L\psi_1=&\sec h(D\eta)^4\left(\frac{-1440D^5\vartheta_0}{A}-\frac{144BD^3\vartheta_0^2}{A^2}\right)\tanh(D\eta)+\sec h(D\eta)^6\\ &\left(\frac{2160D^5\vartheta_0}{A}+\frac{216BD^3\vartheta_0^2}{A^2}+\frac{54D\vartheta_0^3}{A^2}\right)\tanh(D\eta)+\sec h(D\eta)^2\\ &\left(\frac{96D^5\vartheta_0}{A}-\frac{6D\,\vartheta_0\vartheta_1}{A}\right)\tanh(D\eta).\end{aligned} \tag{22}$$

The above equation is a third-order linear differential equation with respect to ψ_1 with inhomogeneous terms in the right-hand side. The homogeneous equation, $L\psi_1=0,$, satisfying the boundary conditions, has a solution that is proportional to sech² (Dη) tanh (Dη). Therefore the secular term exist previously has disappeared on the right-hand side. Let us assume the solution of the above equation in a form

$$\psi_1 = \mu_1 \sec h^2(D\eta) + \mu_2 \sec h^4(D\eta) \tag{23}$$

It is easily observed that the coefficient of sech² (Dη) on the left-hand side cancels out and $L\psi_1$ is expressed in terms of $\sec h^4(D\eta)$ and $\sec h^6(D\eta)$ .Then the coefficient of sech² (Dη) on the right-hand side should vanish, which leads to the correction of the velocity

$$\vartheta_1 = \frac{4\vartheta_0^2}{B^2}. \tag{24}$$

The inhomogeneous equation without secularity is solved, leading to

$$\psi_1 = \frac{-30\ \vartheta_0^2}{AB^2} \sec h^2(D\eta)\ + \frac{45\ \vartheta_0^2}{AB^2} \sec h^4(D\eta). \tag{25}$$

The above solutions, (24) and (25), agree with the solutions obtained by other perturbation methods [23-24].

In the next orders of ε, substituting equation (21),(24) and (25) into (19.c,d), we can followed in the same manner as we obtained ϑ_1 and ψ_1 just above. The results are

$$\vartheta_2 = 0,$$

$$\psi_2 = \frac{90\vartheta_0^3}{AB^4} \sec h^2(D\eta) - \frac{1395\vartheta_0^3}{AB^4} \sec h^4(D\eta) + \frac{1395\vartheta_0^3}{AB^4} \sec h^6(D\eta). \tag{26}$$

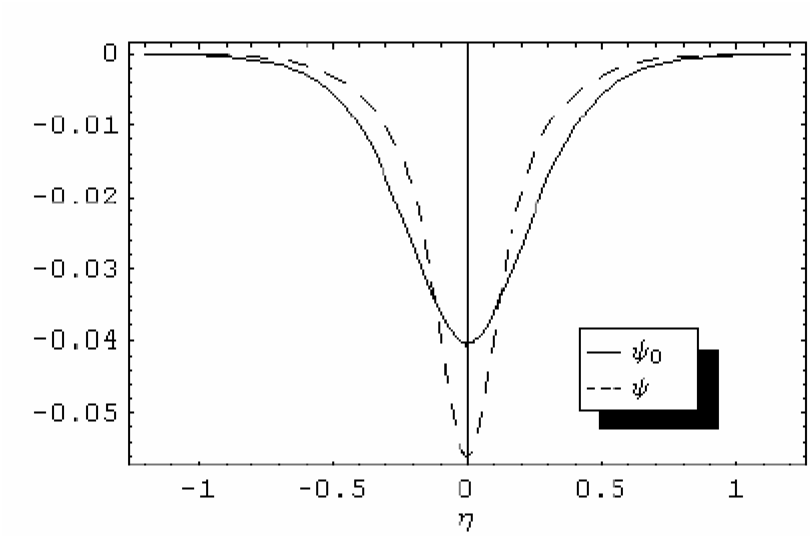
$$\vartheta_3 = 0,$$

$$\psi_3 = \frac{-3708\vartheta_0^4}{AB^6} \sec h^2(D\eta) + \frac{31554\vartheta_0^4}{AB^6} \sec h^4(D\eta) - \frac{99324\vartheta_0^4}{AB^6} \sec h^6(D\eta) + \frac{74493\vartheta_0^4}{AB^6} \sec h^8(D\eta). \tag{27}$$

Combining equations (21), (24), (25), (26) and (27), we obtain a solution of the KdV equation with fifth-order dispersion term. It is worth noting that the higher-order solutions are expressed by power series of the lowest-order solution and that the wave velocity depends only on the first order of ε, while the waveform of a solitary wave depends on all orders of ε.

CONCLUSION AND REMARKS

In this paper, a study of small but finite amplitude EAWs in unmagnetized plasmas containing cold relativistic electron fluid, Maxwellian hot electrons, a relativistic electron beam and stationary ions has been investigated. The basic set of fluid equations is reduced to the well known KdV equation (13) using reductive perturbation theory. As it is well-known, the amplitude of the electron-acoustic solitons as well as parametric regime where the solitons can exist is sensitive to the beam parameters. Moreover, as the wave amplitude increases, the width and the velocity of soliton deviate from the prescribed KdV equation i.e the breakdown of KdV approximation. Naturally, one may ask to what extent the higher-order solution modify the soliton amplitude and width. In other words, is the higher order approximations increase the amplitude and decrease the width of the solitons. Therefore, we directed to suggest an alternative form of KdV equation (16) taking into account the higher order dispersion added as a perturbed term. The solution of the resulting equation has been carried out via a simple technique delivered before by [22]. To make our result physically relevant, numerical studies have been made using plasma parameters close to those values corresponding to the day side auroral zone[5].



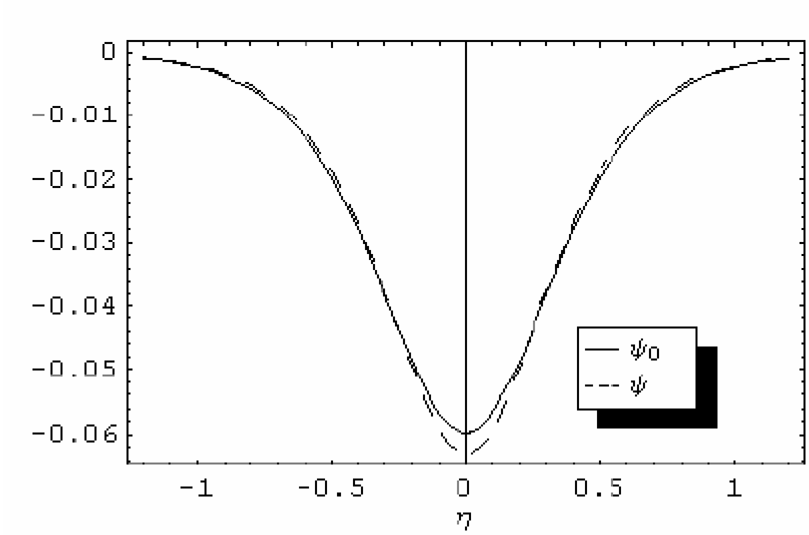
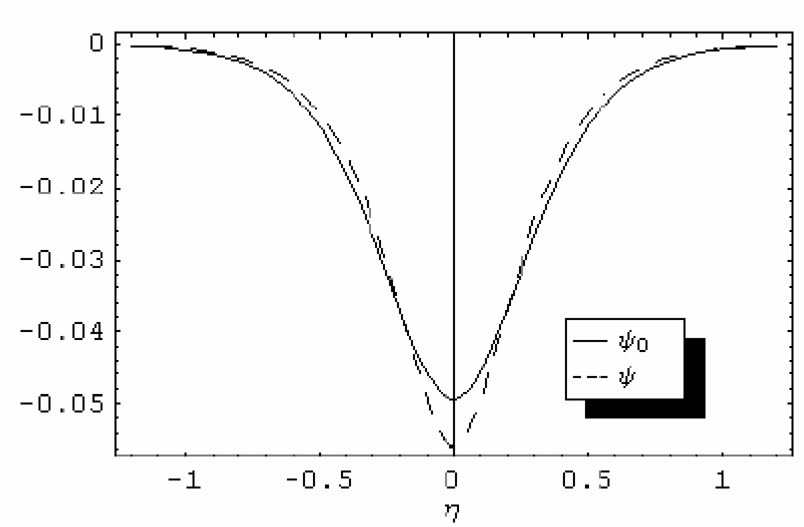


Fig. 1. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=20 , $\frac{u_{b0}}{c} = 0.001$, $\frac{u_{c0}}{c} = 6.6 \times 10^{-4}$, ε=0.00004 and v =1.7

Fig. 2. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=20 , $\frac{u_{b0}}{c} = 0.001$, $\frac{u_{c0}}{c} = 9.99 \times 10^{-4}$, ε=0.00004 and v =1.7.



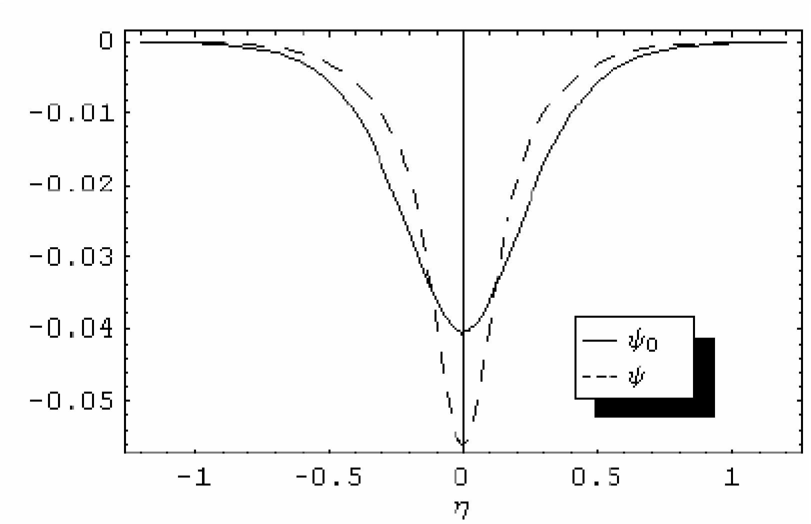
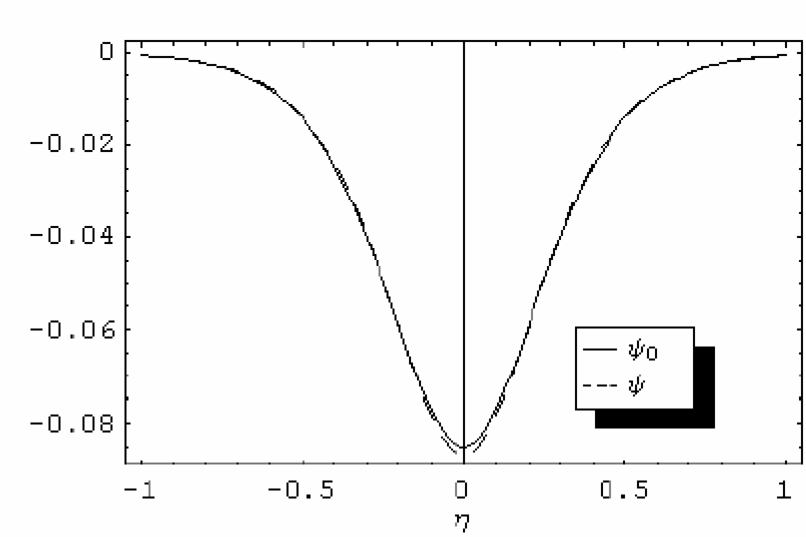


Fig. 3. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=20 , $\frac{u_{b0}}{c} = 1 \times 10^{-3}$, $\frac{u_{c0}}{c} = 0.001$, ε=0.00004 and v =1.7.

Fig. 4. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=20 , $\frac{u_{b0}}{c} = 11.3 \times 10^{-4}$, $\frac{u_{c0}}{c} = 0.001$, ε=0.00004 and v =1.7.



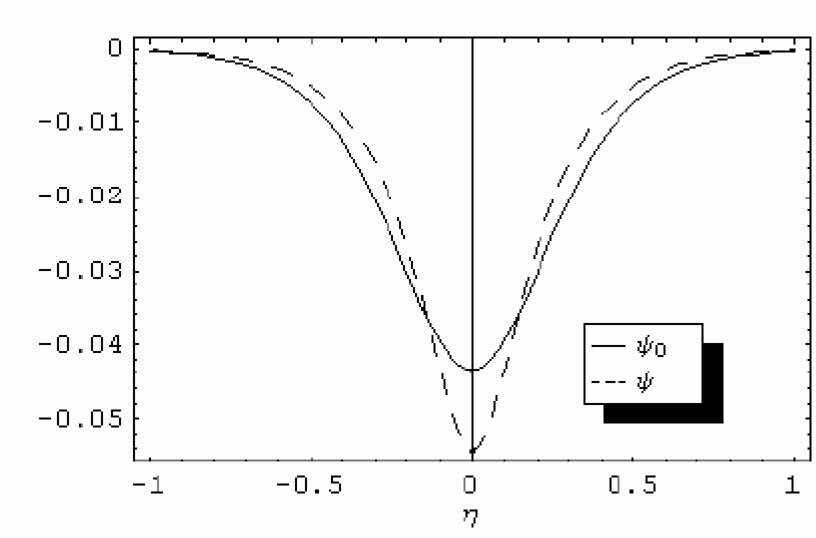
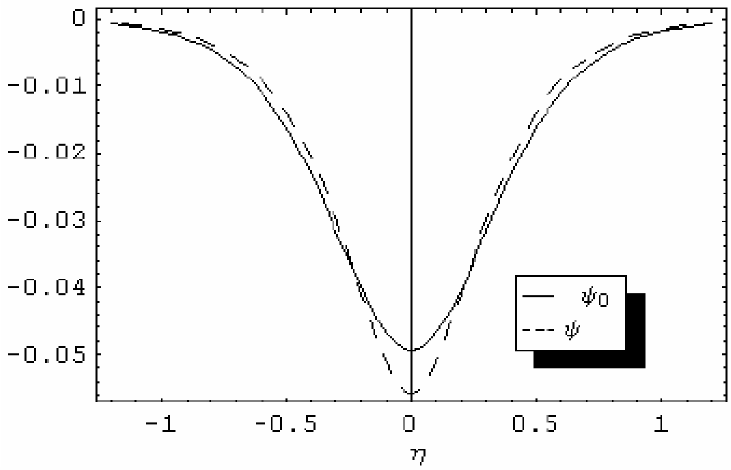


Fig. 5. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=65 , $\frac{u_{b0}}{c} = 0.001$, $\frac{u_{c0}}{c} = 0.001$, ε=0.00004 and v =1.7.

Fig. 6. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=60 , $\frac{u_{b0}}{c} = 0.001$, $\frac{u_{c0}}{c} = 0.001$, ε=0.00004 and v =1.7.



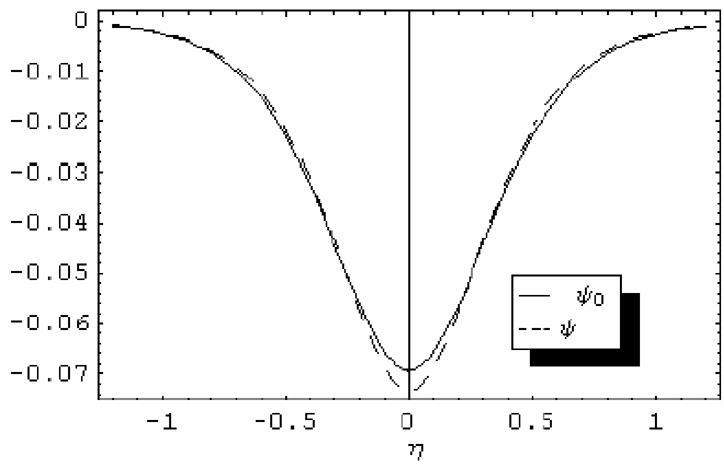


Fig. 7. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=20 , $\frac{u_{b0}}{c} = 0.001$, $\frac{u_{c0}}{c} = 0.001$, ε=0.00004 and v =1.7.

Fig. 8. Plot of ψ_0, ψ against η, Where, $\delta_1 = 0.01$, α=5,β=0.1, σ=10, θ=20 , $\frac{u_{b0}}{c} = 0.001$, $\frac{u_{c0}}{c} = 0.001$, ε=0.00004 and v =1.7.

To examine the effect of higher-order approximation on the amplitude and width of the soliton. For instance, Fig.(1) shows the variation between the nature of the soliton and the cold relativistic electron fluid $\frac{u_{c0}}{c}$, It is clear that the higher correction plays an essential rule due to the increasing of the amplitude and decreasing the width of soliton solution. However, if $\frac{u_{c0}}{c}$ is larger we find from fig. (2) that there is slightly dependences on the higher order correction. On the other hand, fig. (3) displays the relation between soliton characteristic and relativistic electron beam $\frac{u_{b0}}{c}$, it indicates that there is weak dependences for higher order dispersion for small values of $\frac{u_{b0}}{c}$. By increasing $\frac{u_{b0}}{c}$ as shown in fig. (4), we found that a clear difference between soliton solution and its higher order correction. Figs. (5) and (6) show the effect of higher order correction on the relation between relative cold electron temperature θ and nature of the soliton solutions. This effect is no obvious for small values of θ while it is gradually increase as θ increase. Also, for the seek of a better understanding, we examine the effect of higher-order solution to relation between relative beam temperature σ and soliton solutions. Fig.s (7) and (8) show that the effect of higher order solution is clear for small value of σ and decreases as σ increase. Although we have not referred to any specific observations, the application of our model might be particularly interesting in the auroral region where particle acceleration takes place and where electron beams and relativistic effect are a common feature of electron distribution functions.

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