6th Conference on Nuclear and Particle Physics

17-21 Nov. 2007 Luxor, Egypt

ADOMIAN DECOMPOSITION METHOD FOR TRANSIENT NEUTRON TRANSPORT WITH POMRNING-EDDINGTON APPROXIMATION

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Abstract

The time-dependent neutron transport problem is approximated using the Pomraning-Eddington approximation. This approximation is two-flux approximation that expands the angular intensity in terms of the energy density and the net flux. This approximation converts the integro-differential Boltzmann equation into two first order differential equations. The Adomian decomposition method that used to solve the linear or nonlinear differential equations is used to solve the resultant two differential equations to find the neutron energy density and net flux, which can be used to calculate the neutron angular intensity through the Pomraning-Eddington approximation.

Keywords: Transient neutron transport theory, Pomraning-Eddington approximation, integro-differential Boltzmann equation, Adomian decomposition method.

1. INTRODUCTION

Neutron transport theory is concerned with the migration of neutrons through bulk media. Since the probabilities of scattering of neutrons to all directions in a reactor are not equal to each other as in isotropic scattering, the neutrons are migrated anisotropically. The forward and backward scattering of neutrons in a system can be characterized as anisotropic scattering [1]. The particle flux can be obtained as a solution of the Boltzmann transport equation. The

Boltzmann transport equation in its various integro-differential forms can be used to solve problems arising in reactor physics, astrophysics, charged particle plasmas, traffic flow, radiative transport and transport of particles in porous media [2, 3]. A large collection of works exists on the one-speed neutron transport equation (NTE) with isotropic, linearly anisotropic, and strongly anisotropic scattering for slabs, spheres and cylindrical media [4-17].

Analytical and numerical approaches to solve the neutron transport problems have been of interest to the scientific community. Among these approaches, there are the diffusion (Eddington) approximation [4, 5], the spherical harmonics or P_N method [1, 6, 7], finite element method [8, 9], the discrete ordinates method [10], T_N approximation method [11, 12], Jacobi polynomials approximation method [13], the rigorous integral equation model [14, 15] and Mote-Carlo method [16].

Pomraning suggested a modification of the Eddington (diffusion) approximation [17]. We called this approximation as Pomraning-Eddington approximation (PEA). PEA was used to solve the time-independent radiative transfer equation [17-20]. This approximation is a flux-limited approximation, which approximate the particle angular intensity in terms of the energy density and the net flux. Here, we will use the PEA as an approximation to the transient neutron transport equation (TNTE). This approximation changes the TNTE, which is an integro-differential equation for the angular intensity, into two first order linear differential equations in terms of the energy density and net flux. These two first order differential equations will be solved using the Adomian decomposition method (ADM) [21-26].

The ADM had been represented by Adomian [21, 22] and has been modified by Wazwaz [23, 24] and more recently by Luo [25, 26]. This method is useful for obtaining closed form or numerical approximation for a wide class of stochastic and deterministic problems in science and engineering [21-31]. These problems involve algebraic, linear or nonlinear ordinary or partial differential equation, integro-differential, integral and differential delay equations.

The present work is outlined as follows: The time-dependent neutron transport problem is described in section (2). In section (3), the PEA is used to approximate the TNTE into two first order differential equations. The ADM is used in section (4) to solve the two first order differential equations of the PEA. Section (5) contains the discussions and the conclusions on the results.

2. PROBLEM REPRESENTATION

The time-dependent neutron transport through absorbing, anisotropic scattering, homogeneous slab of thickness L, absorption coefficient σ_a, scattering coefficient σ_s and extinction coefficient σ_t (= $\sigma_a + \sigma_s$) can be described by the time-dependent Boltzmann equation in the from [1]

$$\frac{1}{v}\frac{\partial}{\partial t} I(z, \mu, t) + \mu \frac{\partial}{\partial z} I(z, \mu, t) + \sigma_t I(z, \mu, t) = \frac{\sigma_s}{2}\int_{-1}^{1} d\mu' \, p(\mu, \mu') I(z, \mu', t),$$

$$0 \le z \le L, \qquad -1 \le \mu \le 1 \qquad \text{and} \qquad t \ge 0 \tag{1}$$

Here $I(z, \mu, t)$ is the particle intensity at position (z) and time (t) with scattering angle (θ) of cosine (μ) and velocity (v). The scattering phase function $p(\mu, \mu')$ can be represented in terms of the Legendre polynomials $P_n(\mu)$ by the expansion [2]

$$p(\mu, \mu') = \sum_{n=0}^{\infty} a_n \; P_n(\mu)\, P_n(\mu'), \tag{2}$$

where (a_n) are the Legendre polynomial expansion coefficients with $a_0 = 1$. The scattering phase function $p(\mu, \mu')$ can be taken for linear anisotropic scattering as

$$p(\mu, \mu') = 1 + \bar{a}\; \mu\, \mu', \tag{3a}$$

where the linear anisotropic parameter $\bar{a}$ can be calculated in terms of the Legendre expansion coefficients (a_n) through the relation [32]

$$\bar{a} = \sum_{m=0}^{\infty} \frac{(-1)^m\,(2m)!}{2^{2m}\, m!(m+1)!} a_{2m+1}\,. \tag{3b}$$

The considered medium has two boundaries, the left at z = 0 and the right at z = L, respectively. These boundaries have emissivities ε_L and ε_R, diffuse reflectivity coefficients ρ_L^d and ρ_R^d and specular reflectivity coefficients ρ_L^s and ρ_R^s where $\varepsilon_i + \rho_i^d + \rho_i^s = 1$ and i = L or R. The problem has initial condition and boundary conditions

$$I(z, \mu, 0) = I_{in}, \tag{4a}$$

$$I(0, \mu, t) = \varepsilon_L\, I_L + 2\rho_L^d \int_0^1 d\mu'\, \mu'\, I(0, -\mu', t) + \rho_L^s\; I(0, -\mu, t), \;\; t \geq 0, \mu \geq 0, \tag{4b}$$

$$I(L, -\mu, t) = \varepsilon_R\, I_R + 2\rho_R^d \int_0^1 d\mu'\, \mu'\, I(L, \mu', t) + \rho_R^s\; I(L, \mu, t), \; t \geq 0, \mu \geq 0, \tag{4c}$$

where the initial incidence (I_{in}) equal to the incidence on the left boundary of the medium (I_L).

This problem can be represented in dimensionless form by taken the dimensionless time variable $\tau = \sigma_t v t$, dimensionless space variable $x = \sigma_t z$ and dimensionless particle intensity $\Psi(x, \mu, \tau)$ defined by

$$\Psi(x, \mu, \tau) = \frac{(I(z, \mu, t) - I_L)}{(I_R - I_L)}\,. \tag{5}$$

The TNTE in its dimensionless form can be represented as

$$\frac{\partial}{\partial \tau}\Psi(x, \mu, \tau) + \mu \frac{\partial}{\partial x}\Psi(x, \mu, \tau) + \Psi(x, \mu, \tau) = \frac{\omega}{2} \int_{-1}^{1} d\mu'\; p(\mu, \mu')\, \Psi(x, \mu', \tau),$$

$$0 \leq x \leq x_L,\; -1 \leq \mu \leq 1 \qquad \text{and} \qquad \tau \geq 0, \tag{6}$$

where $\omega = \sigma_s/\sigma_t$ is the single scattering albedo. The initial condition and boundary conditions in dimensionless form are given by

$$\Psi(x, \mu, 0) = 0, \tag{7a}$$

$$\Psi(0, \mu, \tau) = \varepsilon_L + 2\rho_L^d \int_0^1 d\mu' \, \mu' \Psi(0, -\mu', t) + \rho_L^s \Psi(0, -\mu, \tau), \tau \geq 0, \mu \geq 0, \tag{7b}$$

$$\Psi(x_L, -\mu, \tau) = 2\rho_R^d \int_0^1 d\mu' \, \mu' \Psi(L, \mu', t) + \rho_R^s \Psi(x_L, \mu, \tau), \tau \geq 0, \mu \geq 0. \tag{7c}$$

The TNTE in its general form is very complicated and is hard to solve analytically and it is time-consuming to solve numerically. Therefore, it is convenient to use approximate techniques to reduce the problem to a mathematical tractable form. In this paper, we will use the PEA [17-20] to simplify the described problem.

3. POMRANING-EDDINGTON APPROXIMATION

The PEA [17-20] is a two-flux model used to simplify the NTE to a mathematical tractable form. This approximation expands the dimensionless angular intensity into two terms as

$$\Psi(x, \mu, \tau) = E(x, \tau)\, e(x, \mu, \tau) + F(x, \tau)\, O(x, \mu, \tau), \tag{8}$$

where the dimensionless energy density $E(x, \tau)$ is defined by

$$E(x, \tau) = \int_{-1}^{1} d\mu \, \Psi(x, \mu, \tau), \tag{9a}$$

while the dimensionless net radiative flux is given by

$$F(x, \tau) = \int_{-1}^{1} d\mu \, \mu \Psi(x, \mu, \tau), \tag{9b}$$

The two functions $e(x, \mu, \tau)$ and $O(x, \mu, \tau)$ are defined as an even and odd functions for the cosine of scattering angle and they are slowly varying functions with space x and time τ. The even and odd functions are normalized by the relations

$$\int_{-1}^{1} d\mu \, e(x, \mu, \tau) = 1 = \int_{-1}^{1} d\mu \, \mu \, O(x, \mu, \tau). \tag{10}$$

Multiplication of Eq. (6) by $P_0(\mu) = 1$ and integration over $\mu \in [-1, 1]$ lead to the partial differential equation

$$\frac{\partial}{\partial \tau} E(x, \tau) + \frac{\partial}{\partial x} F(x, \tau) + \alpha E(x, \tau) = 0, \tag{11a}$$

where $\alpha = 1 - \omega$.

Also, multiplication of Eq. (6) by $P_1(\mu) = \mu$, integration over $\mu \in [-1, 1]$ and using of the PEA give the following partial differential equation for linear anisotropic scattering

$$\frac{\partial}{\partial \tau} F(x, \tau) + \frac{\partial}{\partial x} [D\, E(x, \tau)] + \beta\, F(x, \tau) = 0, \qquad (11b)$$

where $\beta = 1 - \frac{\bar{a}}{3}\,\omega$ and D, which is a slowly varying function for space x and time τ and is called the diffusion parameter, is defined by

$$D = \int_{-1}^{1} d\mu\ \mu^2\, e(x, \mu, \tau). \qquad (12)$$

Substituting the PEA, Eq. (8) into the initial and boundary conditions, Eqs (7) multiplying, respectively by 1 and μ and integrating over $\mu \in [-1, 1]$ lead to

$$E(x, 0) = 0, \qquad (13a)$$

$$E(0, \tau) = E_{00} = 2\frac{\varepsilon_L (1+\rho_L^s)(1-O_0)}{(2-\varepsilon_L)(1-\rho_L^s) - 4e_1[2\rho_L^d + \varepsilon_L (1+\rho_L^s) O_0]}, \qquad (13b)$$

$$E(x_L, \tau) = 0, \qquad (13c)$$

$$F(x, 0) = 0, \qquad (14a)$$

$$F(0, \tau) = F_{00} = \frac{\varepsilon_L (1-\rho_L^s)(1-4e_1)}{(2-\varepsilon_L)(1-\rho_L^s) - 4e_1[2\rho_L^d + \varepsilon_L (1+\rho_L^s) O_0]}, \qquad (14b)$$

$$F(x_L, \tau) = 0, \qquad (14c)$$

which are the initial and boundary conditions of Eqs (11a) and (11b) for the energy density $E(x, \tau)$ and the net flux $F(x, \tau)$. The functions O_0 and e_1 are given by

$$O_0 = \int_0^1 d\mu\ O(x, \mu, \tau). \qquad (15a)$$

$$e_1 = \int_0^1 d\mu\ \mu\, e(x, \mu, \tau). \qquad (15b)$$

The even and odd functions $e(x, \mu, \tau)$ and $O(x, \mu, \tau)$ in the PEA can be calculated for linear anisotropic scattering in homogeneous medium as follows. Substituting with PEA, Eq. (8), into Eq. (6) for linear anisotropic scattering, separation of even terms and odd terms of μ and using of Eqs (11a & b) for $\partial E(x, \tau)/\partial \tau$ and $\partial F(x, \tau)/\partial \tau$ lead to

$$e(x, \mu, \tau) = \frac{\omega}{2R_2} \frac{(R_2 + \bar{a}\, f_1 \upsilon^2 \mu^2)}{f_1 (1-\upsilon^2 \mu^2)}, \qquad (16a)$$

$$O(x, \mu, \tau) = \frac{\omega}{2R_2} \frac{\upsilon^2}{R_1} \frac{(R_2 + \bar{a} f_1)}{(1-\upsilon^2 \mu^2)}, \qquad (16b)$$

where

$$R_1 = \frac{-\partial F/\partial x}{E(x,\tau)}, \tag{17a}$$

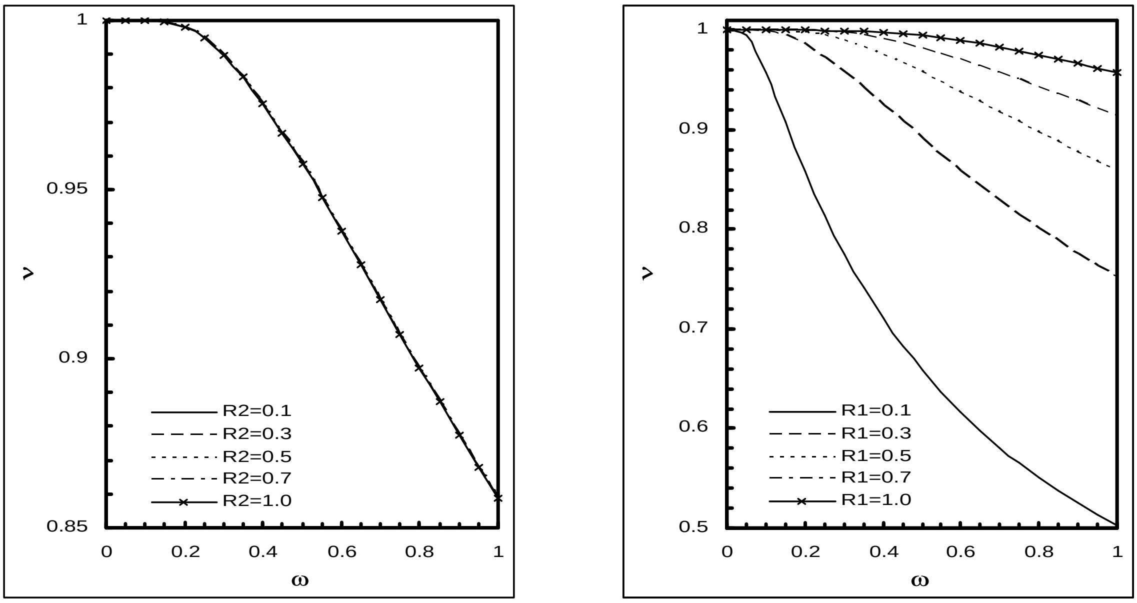
$$R_2 = \frac{-\partial E/\partial x}{F(x,\tau)}, \tag{17b}$$

$$f_1 = R_1 + \omega, \tag{17c}$$

$$f_2 = D\,R_2 + \frac{\bar{a}}{3}\,\omega, \tag{17d}$$

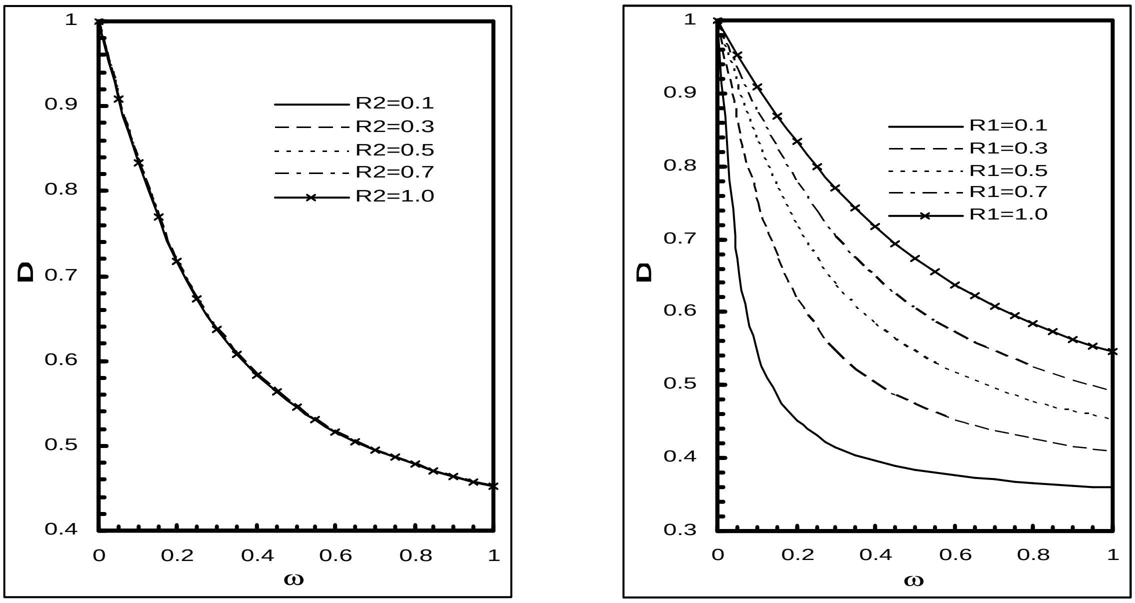
and

$$\nu^2 = \frac{R_1}{f_1}\,\frac{R_2}{f_2}. \tag{18}$$



(a) (b)

Fig 1. Relation between the albedo (ω) and the flux limiter (ν) for isotropic scattering, homogeneous medium for: (a) constant value of R_1=0.5 and different values of R_2, (b) constant value of R_2=0.5 and different values of R_1.



(a) (b)

Fig 2. Relation between the albedo (ω) and the diffusion parameter (D) for isotropic scattering in homogeneous medium for: (a) constant value of R_1=0.5 and different values of R_2, (b) constant value of R_2=0.5 and different values of R_1.

The parameters R_1 and R_2, which are the ratios between the energy density E(x, τ) and the net flux F(x, τ) and their space variations can be called flux-limited parameters.

The calculations of both e(x, μ, τ) and O(x, μ, τ) are carried out using the assumption that they are slowly varying functions for both x and τ or using the more reliable assumptions

$$\frac{\partial}{\partial \tau} e(x, \mu, \tau) + \mu \frac{\partial}{\partial x} e(x, \mu, \tau) = 0, \tag{19a}$$

$$\frac{\partial}{\partial \tau} O(x, \mu, \tau) + \mu \frac{\partial}{\partial x} O(x, \mu, \tau) = 0, \tag{19b}$$

which lead to $\partial D(x, \tau)/\partial \tau = \partial D(x, \tau)/\partial x = 0$, i. e; the diffusion parameter D is a constant and does not depend on both the space and time. The diffusion parameter D can be represented by substituting Eq. (16a) into the definition of Eq. (12) to give

$$D = \frac{R_1}{f_1 \nu^2} - \frac{\bar{a}}{3} \frac{\omega}{R_2}. \tag{20}$$

Here the parameter ν can be calculated using the following transcendental equation

$$\nu = \frac{1 - \exp(-2\nu Y / \omega)}{1 + \exp(-2\nu Y / \omega)}, \tag{21a}$$

where

$$Y = f_1 \frac{(R_2 + \bar{a}\,\omega)}{(R_2 + \bar{a}\,f_1)}. \tag{21b}$$

This transcendental equation is given by substituting any of the forms of $e(x, \mu, \tau)$ or $O(x, \mu, \tau)$ of Eqs (16) into Eq. (10). The parameter ν can be calculated for each group of values of the albedo ω, the linear anisotropic scattering parameter $\bar{a}$ and the flux-limited parameters R_1 and R_2 numerically using an iteration method.

The energy density $E(x, \tau)$ and the net flux $F(x, \tau)$ can be calculated by solving the partial differential equations, Eqs (11), with the initial and boundary conditions of Eqs (13) and (14). The ADM [21-26] will be used in this paper to solve the partial differential equations, Eqs. (11).

4. SOLUTION USING ADOMIAN DECOMPOSITION METHOD

The ADM had been represented by Adomian [21, 22] and modified by Wazwaz [23, 24] and more recently by Luo [25, 26]. This method is useful for obtaining closed form or numerical approximation for a wide class of stochastic and deterministic problems in science and engineering [21-31].

The ADM will be used here to solve the couple of differential equations (11a & b) with the initial and boundary conditions given by Eqs (13) and (14). These equations represent ill-posed problem due to their inhomogeneous boundary conditions. Therefore, the solution of these equations can be represented by [25, 26]

$$E(x, \tau) = E_0(x, \tau) + W_0(x, \tau), \tag{22a}$$

$$F(x, \tau) = F_0(x, \tau) + W_1(x, \tau), \tag{22b}$$

where

$$W_0(x, \tau) = (1 - \frac{x}{x_L})\, E(0, \tau) + \frac{x}{x_L}\, E(x_L, \tau) = (1 - \frac{x}{x_L})\, E_{00}, \tag{23a}$$

$$W_1(x, \tau) = (1 - \frac{x}{x_L})\, F(0, \tau) + \frac{x}{x_L}\, F(x_L, \tau) = (1 - \frac{x}{x_L})\, F_{00}. \tag{23b}$$

Here the functions $E_0(x, \tau)$ and $F_0(x, \tau)$ are described by the differential equations

$$\frac{\partial}{\partial \tau} E_0(x, \tau) + \frac{\partial}{\partial x} F_0(x, \tau) + \alpha\, E_0(x, \tau) = H_0(x, \tau), \tag{24a}$$

$$\frac{\partial}{\partial \tau} F_0(x, \tau) + D \frac{\partial}{\partial x} E_0(x, \tau) + \beta\, F_0(x, \tau) = H_1(x, \tau), \tag{24b}$$

where

$$H_0(x, \tau) = - \frac{\partial}{\partial x} W_1(x, \tau) - \alpha\, W_0(x, \tau) = \frac{1}{x_L} F_{00} - \alpha\, (1 - \frac{x}{x_L})\, E_{00}, \tag{25a}$$

$$H_1(x, \tau) = - D \frac{\partial}{\partial x} W_0(x, \tau) - \beta\, W_1(x, \tau) = \frac{D}{x_L} E_{00} - \beta\, (1 - \frac{x}{x_L})\, F_{00}. \tag{25b}$$

The partial differential equations (24a) and (24b) can be rewritten in the operator form

$$\hat{L}_\tau\, E_0(x, \tau) = H_0(x, \tau) - \alpha\, E_0(x, \tau) - \hat{L}_x\, F_0(x, \tau), \tag{26a}$$

$$\hat{L}_\tau\, F_0(x, \tau) = H_1(x, \tau) - \beta\, F_0(x, \tau) - D\, \hat{L}_x\, E_0(x, \tau), \tag{26b}$$

where the differential time operator $\hat{L}_\tau$ and the differential space operator $\hat{L}_x$ are defined by

$$\hat{L}_\tau = \frac{\partial}{\partial \tau}, \qquad \text{and} \qquad \hat{L}_x = \frac{\partial}{\partial x}, \tag{27}$$

while the inverse time operator is defined by

$$\hat{L}_\tau^{-1} = \int_0^\tau d\tau'. \tag{28}$$

These last two partial differential equations have homogeneous boundary conditions for both $E_0(x, \tau)$ and $F_0(x, \tau)$, while the initial conditions of these equations are

$$E_0(x, 0) = - W_0(x, 0) = - (1 - \frac{x}{x_L})\, E_{00}, \tag{29a}$$

$$F_0(x, 0) = - W_1(x, 0) = - (1 - \frac{x}{x_L})\, F_{00}, \tag{29b}$$

where E_{00} and F_{00} are represented by Eqs (13b) and (14b).

The solution of Eqs (26a) and (26b) using the ADM is built by assuming $E_0(x, \tau)$ and $F_0(x, \tau)$ as series of the forms [21]

$$E_0(x, \tau) = \sum_{n=0}^{\infty} \varepsilon_n(x, \tau), \tag{30a}$$

$$F_0(x, \tau) = \sum_{n=0}^{\infty} \phi_n(x, \tau), \tag{30b}$$

Substituting by these series expansions into Eqs (26a) and (26b) and affecting with the inverse time operator lead to

$$\sum_{n=0}^{\infty}\varepsilon_n(x,\tau) = E_0(x, 0) + \hat{L}_{\tau}^{-1} H_0(x, \tau) - \hat{L}_{\tau}^{-1}\{\alpha \sum_{n=0}^{\infty}\varepsilon_n(x,\tau) + \hat{L}_x \sum_{n=0}^{\infty}\phi_n(x,\tau)\}, \tag{31a}$$

$$\sum_{n=0}^{\infty}\phi_n(x,\tau) = F_0(x, 0) + \hat{L}_{\tau}^{-1} H_1(x, \tau) - \hat{L}_{\tau}^{-1}\{\beta \sum_{n=0}^{\infty}\phi_n(x,\tau) + D\,\hat{L}_x \sum_{n=0}^{\infty}\varepsilon_n(x,\tau)\}, \tag{31b}$$

The ADM gives the following recurrence relations to calculate the expansion terms as

$$\varepsilon_0(x, \tau) = E_0(x, 0) + \hat{L}_{\tau}^{-1} H_0(x, \tau), \tag{32a}$$

$$\phi_0(x, \tau) = F_0(x, 0) + \hat{L}_{\tau}^{-1} H_1(x, \tau), \tag{32b}$$

$$\varepsilon_{n+1}(x, \tau) = - \hat{L}_{\tau}^{-1}\{\alpha\, \varepsilon_n(x, \tau) + \hat{L}_x\, \phi_n(x, \tau)\}, \qquad n \geq 0, \tag{33a}$$

$$\phi_{n+1}(x, \tau) = - \hat{L}_{\tau}^{-1}\{\beta\, \phi_n(x, \tau) + D\, \hat{L}_x\, \varepsilon_n(x, \tau)\}, \qquad n \geq 0. \tag{33b}$$

Substituting Eqs (25a & b) and (29a & b) into Eqs (32a & b) leads to

$$\varepsilon_0(x, \tau) = \frac{1}{x_L} F_{00}\, \tau - (1 - \frac{x}{x_L})\, E_{00}\, (1 + \alpha\tau), \tag{34a}$$

$$\phi_0(x, \tau) = \frac{D}{x_L} E_{00}\, \tau - (1 - \frac{x}{x_L})\, F_{00}\, (1 + \beta\tau). \tag{34b}$$

These zero-order terms are ill-defined functions and the final solutions using ADM will not satisfy the boundary conditions of the problem. Therefore, these terms can be rewritten as Fourier expansions of the forms

$$\varepsilon_0(x, \tau) = \sum_{m=0}^{\infty} a_m(\tau) \sin(C_m\, x), \tag{35a}$$

$$\phi_0(x, \tau) = \sum_{m=0}^{\infty} b_m(\tau) \cos(C_m\, x), \tag{35b}$$

where

$$a_m(\tau) = (1 + \alpha\tau)\, A_m - \tau\, C_m\, B_m, \tag{36a}$$

$$b_m(\tau) = (1 + \beta\tau)\, B_m, \tag{36b}$$

$$A_m = - \frac{2}{m\pi} E_{00}, \tag{37a}$$

$$B_m = \frac{2}{(m\pi)^2}[(-1)^m - 1]\, F_{00}, \tag{37b}$$

$$C_m = \frac{m\pi}{x_L}. \tag{38}$$

Substituting with Eqs (35)-(38) into Eqs (33a & b) leads to

$$\varepsilon_0(x, \tau) = \sum_{m=0}^{\infty} \{(1 + \alpha\tau)\, A_m - \tau\, C_m\, B_m\} \sin(C_m\, x), \tag{39a}$$

$$\phi_0(x, \tau) = \sum_{m=0}^{\infty} \{(1 + \beta\tau)\, B_m,\} \cos(C_m\, x), \tag{39b}$$

$$\varepsilon_1(x, \tau) = \sum_{m=0}^{\infty} \{-[\alpha\tau + \alpha^2 \frac{\tau^2}{2!}]\, A_m + [\tau + (\alpha+\beta)\frac{\tau^2}{2!}]\, C_m\, B_m\} \sin(C_m x), \tag{40a}$$

$$\phi_1(x, \tau) = \sum_{m=0}^{\infty} \{-(\tau + \alpha\frac{\tau^2}{2!})\, DC_m A_m - [\beta\tau + \beta^2\frac{\tau^2}{2!} - DC_m^{\ 2}\frac{\tau^2}{2!}]\, B_m\}\cos(C_m\, x). \tag{40b}$$

$$\varepsilon_2(x, \tau) = \sum_{m=0}^{\infty} \{[\alpha^2\frac{\tau^2}{2!} + \alpha^3\frac{\tau^3}{3!} - (\frac{\tau^2}{2!} + \alpha\frac{\tau^3}{3!})\, DC_m^{\ 2}]\, A_m$$

$$- [(\alpha+\beta)\frac{\tau^2}{2!} + (\alpha^2+\alpha\beta+\beta^2)\frac{\tau^3}{3!} - \frac{\tau^3}{3!}\, DC_m^{\ 2}]\, C_m B_m\} \sin(C_m x), \tag{41a}$$

$$\phi_2(x, \tau) = \sum_{m=0}^{\infty} \{(\alpha+\beta)(\frac{\tau^2}{2!} + \alpha\frac{\tau^3}{3!})\, DC_m\, A_m$$

$$+ [\beta^2\frac{\tau^2}{2!} + \beta^3\frac{\tau^3}{3!} - (\frac{\tau^2}{2!} + (\alpha+2\beta)\frac{\tau^3}{3!})\, DC_m^{\ 2}]\, B_m\} \cos(C_m\, x). \tag{41b}$$

$$\varepsilon_3(x, \tau) = \sum_{m=0}^{\infty} \{[-\alpha^3\frac{\tau^3}{3!} - \alpha^4\frac{\tau^4}{4!} + (2\alpha+\beta)(\frac{\tau^3}{3!} + \alpha\frac{\tau^4}{4!})\, DC_m^{\ 2}]\, A_m$$

$$[(\alpha^2+\alpha\beta+\beta^2)\frac{\tau^3}{3!} + (\alpha^3+\alpha^2\beta+\alpha\beta^2+\beta^3)\frac{\tau^4}{4!} - (\frac{\tau^3}{3!} + 2(\alpha+\beta)\frac{\tau^4}{4!})\, DC_m^{\ 2}]\, C_m B_m\}\sin(C_m x), \tag{42a}$$

$$\phi_3(x, \tau) = \sum_{m=0}^{\infty} \{[-(\alpha^2+\alpha\beta+\beta^2)(\frac{\tau^3}{3!} + \alpha\frac{\tau^4}{4!})DC_m + (\frac{\tau^3}{3!} + \alpha\frac{\tau^4}{4!})D^2C_m^{\ 4}]\, A_m$$

$$- [\beta^3 \frac{\tau^3}{3!} + \beta^4 \frac{\tau^4}{4!} \quad - ((\alpha+2\beta) \frac{\tau^3}{3!} + (\alpha^2+2\alpha\beta+3\beta^2) \frac{\tau^4}{4!})DC_m^{\,2} + \frac{\tau^4}{4!} D^2C_m^{\,4}]\, B_m\}$$

$$\cos(C_m\, x). \tag{42b}$$

The other terms of the expansions are calculated using the recurrence relations of Eqs (33) with the Maple package. The sum to infinity of the calculated terms leads to

$$E_0(x, \tau) = -2 \frac{E_{00}}{x_L} \sum_{m=1}^{\infty} \frac{\sin(C_m x)}{C_m} \{1 +$$

$$\sum_{k=1}^{\infty} \frac{(-DC_m^2)^k}{[(k-1)!]^2} \sum_{j=0}^{\infty} \frac{(-\tau)^{j+2k}}{(j+2k)!} \sum_{i=0}^{j} \frac{(k+j-i-1)!}{(j-i)!} \frac{(k+i-1)!}{i!} \alpha^{j-i} \beta^i \}, \tag{43a}$$

$$F_0(x, \tau) = \frac{x}{x_L} F_{00} - \alpha x \frac{(2x_L - x)}{2x_L} E_{00} + 2 \frac{E_{00}}{x_L} \sum_{m=1}^{\infty} \frac{[1-\cos(C_m x)]}{C_m^2} \{\alpha + \sum_{k=1}^{\infty} \frac{(-DC_m^2)^k}{[(k-1)!]^2}$$

$$\sum_{j=0}^{\infty} \frac{(-\tau)^{j+2k}}{(j+2k)!} [\frac{(j+2k)}{\tau} + \alpha] \sum_{i=0}^{j} \frac{(k+j-i-1)!}{(j-i)!} \frac{(k+i-1)!}{i!} \alpha^{j-i} \beta^i \}. \tag{43b}$$

Substituting these equations into Eqs (22a & b) using Eqs (23a & b) lead to the expansion forms of the dimensionless energy density $E(x, \tau)$ and the dimensionless net flux $F(x, \tau)$. The resultant relations with Eqs (16a & b) can be used to calculate dimensionless angular intensity $\Psi(x, \mu, \tau)$ using the PEA, Eq (8).

5. DISCUSSION AND CONCLUSION

The time-dependent Boltzmann equation is used to describe the migration of neutrons through a bulk medium. The neutron migration is considered as an anisotropic scattering and absorption in a homogeneous medium. The medium is considered to have diffuse and specular reflecting boundaries.

The TNTE in its integro-differential form is hard to be solved analytically and it is time-consuming to be solved numerically. It is convenient to simplify the Boltzmann integro-differential equation using some approximations. Here, the PEA is used to simplify the neutron migration problem described by the Boltzmann equation. This approximation transforms the integro-differential Boltzmann equation into two first order partial differential equations for the energy density and net flux.

The ADM is used in this paper to solve the couple first order differential equations to find energy density and the net flux of the neutrons. This method leads to semi-closed forms of both the energy density and the net flux.

The solution of the transcendental equation, Eq (21), as a relation between the flux limiter parameter (ν) and the single scattering albedo (ω) is represented in Fig 1. The relation between the diffusion parameter (D) and the single scattering albedo (ω) is represented in Fig 2. The two figures, Fig. 1 and 2, show that the flux-limited parameter (R_1) has affects to change the flux limiter parameter (ν) and the diffusion parameter (D) while the flux-limited parameter (R_2) has no effects on these two parameters. This means that the flux-limited parameter (R_2) can be taken as a unity. i. e; $F(x, \tau) = -\partial E(x, \tau)/\partial x$, which is Fick's law.

E(x. τ)

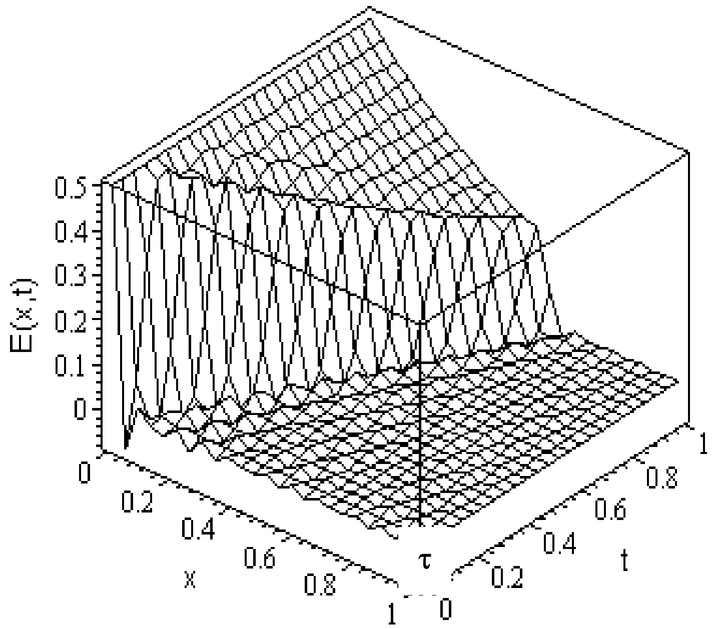


Fig 3. The neutron density in a homogeneous, pure absorbing medium in a slab of unit thickness with transparent boundaries.

F(x. τ)

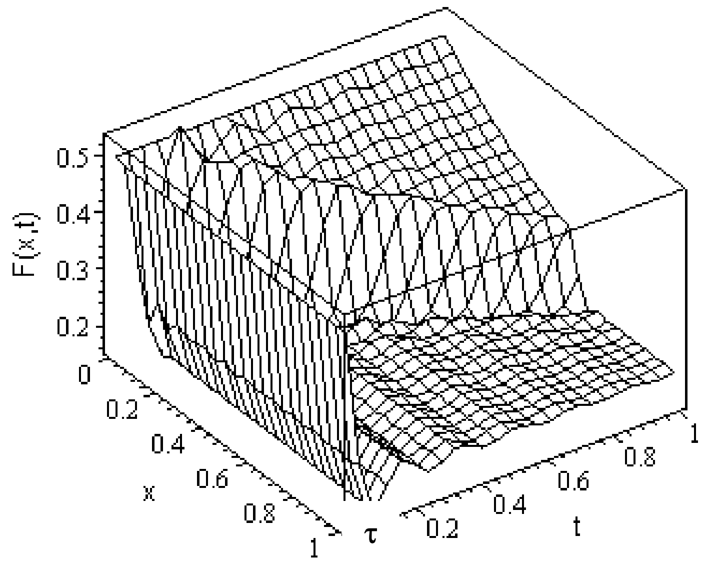


Fig 4. The neutron net flux in a homogeneous, pure absorbing medium in a slab of unit thickness with transparent boundaries.

The neutron density E(x, τ) is represented as a 3-dimensions graph in Fig 3, while Fig 4 has a 3-dimensions representation of the neutron net flux. The calculations in the two figures 3 and 4 are carried out for a homogeneous slab of thickness x_L = 1. The slab has pure absorbing medium, $\omega_0 = 0$ with transparent boundaries, $\rho_i^d = \rho_i^s = 0$, i = L and R.

ACKNOWLEDGMENT

The authors would like to pass their great appreciation to their **Prof. Said A. El-Wakil** for his suggestions, encouragement, and review of their works at all.

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