Kaon-Nucleon Interaction in One-Boson-Exchange Picture at Intermediate Energies

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On the basis of One-Boson-Exchange-Potential (OBEP) picture, it is derived and suggested for use a kaon-nucleon (K^+N) potential in the energy region $P_{lab} < 1\ GeV$ based on the exchange of three/four mesons, one attractive scalar $\sigma(0^+, 0)$ meson and three repulsive vector $\rho(1^-, 1), \omega(1^-, 1)$, and σ_0 mesons in the Dirac space. This structure for the K^+N interaction is consistent with the fact that more additional repulsion is required by the data where the shortest range ω-meson is not prepared to carry such load by blowing up its coupling constant which is restricted to its SU(6) group value. Alternatively, it is proposed to use the phenomenological σ_0- meson of much shorter range and higher mass. Moreover, the derived form of the (K^+N) potential $V(r)$ takes into account the center of mass effect of the two particles of different masses. This is due to the assumption that the interacting particles move under the influence of a harmonic oscillator which in turn enables to deal with the two-body wave function as a product of separate relative and center of mass coordinates wave functions and the known generalized Talmi-Moshinsky-Smirnov (GTMS) brackets for particles with different masses. The radial behavior of the constructed (K^+N) OBEP potential is presented.

Keywords: Kaon-Nucleon interaction, Kaon-Nucleus interaction, One-Boson-Exchange-Potential (OBEP), Microscopic scattering theory

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I. Introduction

Although some phenomenological trials has been made to use some pictures for the quark- gluon interaction,however due to the absence of a rigorous sharp picture for this interaction and also the difficulties of the extrapolation of the informations coming from different energy domains to be incorporated in a natural way in one theory [1], it is inevitable to use baryons and mesons which are considered the suitable and proper variables and in the same time they are the collective degrees of freedom of the quantum chromodynamics (QCD) theory at low and intermediate energy regions. Moreover, the meson theory of the strong nuclear force has been vigorously developed to describe these interactions in terms of few numbers of parameters and the Bonn-potential, either charge-dependent or independent [2,3], is one, between others, of the distinguished and very useful models to describe the NN interaction and also the meson-baryon, baryon-antibaryon, and meson-meson interaction after its concepts has been extended to such interactions [4]. In this study, along the same guide lines of J*ü*lich group-potential, we construct and suggest for use the OBEP picture of the K^+N potential at intermediate energy range $P_{lab} < 1\ GeV$. The model is consistent with our believe that the contributions from higher order kernels are minimized by the nature of K^+N interaction and the dynamics can be extended consistently to other processes. This semi-relativistic K^+N potential can serve as a corner

stone in a microscopically K^+A optical potentials. We propose to describe the dynamics of the K^+N interaction in the energy range $E_{lab} < 1\ GeV$ to be mediated by attractive $\sigma(0^+, 0)$ and repulsive $\rho(1^-, 1), \omega(1^-, 0)$ mesons. An additional phenomenological repulsive σ_0 meson of much shorter range than the ω-meson and of higher mass is introduced to account for the additional repulsion obviously required by the data.
Indeed, there is much initial enthusiasm for the study of $K^\pm$ reactions with nuclei. This provided a considerable stimulus not only for the study of the different nuclear density regions these particles can probe, but also towards a better understanding of the reaction mechanisms. Definite inherent features favor the kaons as distinguished hadronic probes for studying the fine peculiarities of the intermediate energy nuclear reactions and the important signals may be gained from this energy region. In fact, they transfer to the nucleus a new degree of freedom i.e. strangeness a quantum number believed to be conserved in strong interactions. Due to the quark structure of kaons ($K^+ \equiv u\overline{s}, K^- \equiv \overline{u}s, and\ K^0 \equiv d\overline{s}$), the interaction between kaons and nuclei is rather weak and this weakness reflects in the large mean free path λ of K^+ and also relatively of K^- in nuclear matter (e.g.,λ= 5-7 fm for K^+ at $P_{lab} \leq 0.8\ GeV/c$). However,the K^--meson, as some other hadronic probes do, interacts with nuclei more peripherally and can not be used to sound their inner structure, while the K^+-meson is indeed capable to penetrate in the interior of the nucleus and probe entirely its volume and "see" some exotics in the deeper nuclear levels. The main interest in K^- projectile arises from its use in the production of hypernuclei using $(K^-, \pi^\pm)$ reaction. Further, very different K^+ and K^- nuclear interactions are observed where the K^- data are better reproduced than those of the K^+ even though the latter is expected to have simple reaction mechanism [5].
Our principal motivation for such study is to open up the possibility for a well defined discussion of the medium effects in such soft intermediate energy K^+A interactions. Second principal purpose is to demonstrate that the meson exchange picture can serve as basis of successful phenomenology of intermediate energy K^+N and should be a good starting point for realistic microscopic K^+A reactions. In section two we elucidate the mathematical formulation of the problem. Section three is devoted to the parameterization while in Sec.4 the results and their discussion and some conclusions are given.

II. Mathematical Formulation

From both experimental and phenomenological point of view a reasonable fit of the K^+N data indicates that K^+N interaction is rather short ranged, the fact which reflects the importance of the vector-isovector $\rho(1^-, 1)$ and the vector-isoscalar $\omega(1^-, 0)$ mesons exchanges in this interaction. However, because of the intimate interplay between the repulsive ω- and attractive σ-exchange which is usually responsible for the strong inherent cancellation between these fields, it is proposed to include the broad scaler-isoscalar $\sigma(0^+, 0)$ boson in K^+N interaction to report effectively for the 2π-exchange in the S-channel. So, the K^+N interaction proposed in the framework of the OBEP model is to be mediated mainly, in the momentum range less than 1 GeV/c, by (σ, ρ, ω) mesons. Then schematically we can express the K^+N interaction in the form,

$$V_{ki}^{(1)}(r) = V_\sigma(r) + V_\rho(r) + V_\omega(r) \tag{1}$$

where $V_\sigma(r) = -\gamma_1^0\gamma_2^0 J_\sigma(r)$, $V_\rho(r) = \gamma_1^0\gamma_2^0\gamma_1^\mu\gamma_2^\mu J_\rho(r)$, $V_\omega(r) = \gamma_1^0\gamma_2^0\gamma_1^\mu\gamma_2^\mu J_\omega(r)$, while γ_i^0 and

γ_i^μ $(i = 1, 2)$ are the usual Dirac matrices and $J_\sigma(r)$, $J_\rho(r)$, $J_\omega(r)$ are suitable Yukawa-type functions. However, towards a realistic description of the data a more additional repulsion, than obtained by the shortest range ω-meson exchange, which based on the symmetry values is required (see e.g. [6]). Moreover, the need to blow up the ωKN coupling constant to account for the additional repulsion, obviously required by the data, indicates that the ω-meson must carry a load for which it is not prepared. Alternatively, it is proposed a phenomenological repulsive σ_0-meson of much shorter range and higher mass. Then the proposed K^+N potential gets the form,

$$V_{ki}^{(2)}(r) = V_\sigma(r) + V_\rho(r) + V_\omega(r) + V_{\sigma_0}(r) \tag{2}$$

where $V_{\sigma_0}(r) = \gamma_1^0 \gamma_2^0 J_{\sigma_0}(r)$ is taken as in σ-exchange with opposite sign and heavier exchange mass [6].

II.1 Wave Functions Normalizations

In Dirac space the normalization condition for the nucleon wave functions $f_\gamma(\mathbf{r})$ can be written as follows,

$$\langle f_\gamma(\mathbf{r}) | f_\gamma(\mathbf{r}) \rangle = \langle \varphi_\gamma(\mathbf{r}) | \varphi_\gamma(\mathbf{r}) \rangle + \langle \chi_\gamma(\mathbf{r}) | \chi_\gamma(\mathbf{r}) \rangle = \mathbf{1} \tag{3}$$

where $\varphi_\gamma(\mathbf{r})$ and $\chi_\gamma(\mathbf{r})$ are the large and small wave function components respectively. Consequently, the normalized nucleon wave function can be expressed in the form,

$$|\varphi'(\mathbf{r})\rangle = \frac{1}{\sqrt{[1 + (P_2^2/4M_2^2c^2)]}} |\varphi(\mathbf{r})\rangle, \tag{4}$$

where M_2 is the nucleon mass and $\mathbf{P_2}$ is its relative momentum. In case of kaon wave function $f_\alpha(\mathbf{r})$ and due to the zero spin of this particle in addition to the decoupling technique of the Dirac equation we follow here and also the use of the first term only in the expansion of the small wave function component $\chi(\mathbf{r})$ in terms of the large one $\varphi(\mathbf{r})$ as given by the relation,

$$\chi(\mathbf{r}) = \frac{\mathbf{1}}{\mathbf{E} + \mathbf{Mc^2}}[\mathbf{1} + \frac{\mathbf{V}(\mathbf{r})}{\mathbf{E} + \mathbf{Mc^2}} + ...]\mathbf{c}(\sigma.\mathbf{p})\varphi(\mathbf{r}) \tag{5}$$

we find that the kaon wave functions $f_\alpha(\mathbf{r})$ are normalized.

II.2 Coordinates and Momenta for Nucleon-Nucleon and Kaon-Nucleon in Relative and C.M Systems

Here, we follow the notation given in Ref.[7]where for the NN system the relative and C.M. coordinate values are as follows:

$$\mathbf{r} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(\mathbf{r_1} - \mathbf{r_2}) \quad ; \quad \mathbf{R} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(\mathbf{r_1} + \mathbf{r_2}) \tag{6}$$

and the corresponding relative $\mathbf{P_{ij}}$ and center of mass $\mathbf{P_R}$ momenta of the two nucleons can be written as,

$$\mathbf{P_{ij}} \equiv \mathbf{P_r} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{p_i} - \mathbf{p_j}) \quad ; \quad \mathbf{P_R} = \mathbf{p_i} + \mathbf{p_j} \tag{7}$$

While for KN system we denote M_k as the kaon mass and M_j is the nucleon mass, the corresponding coordinate values are as follows,

$$\mathbf{r} = \sqrt{2} \quad \frac{\mathbf{M_k r_1 - M_j r_2}}{\mathbf{M_k + M_j}} \qquad ; \qquad \mathbf{R} = \sqrt{2} \quad \frac{\mathbf{M_k r_1 + M_j r_2}}{\mathbf{M_k + M_j}} \tag{8}$$

Similarly, the corresponding values of the relative $\mathbf{P_{kj}}$ and center of mass KN system momenta get the forms,

$$\mathbf{P_{kN}} \equiv \mathbf{P'_r} = \frac{\mathbf{M_j P_k - M_k P_j}}{\mathbf{M_k + M_j}} \qquad ; \qquad \mathbf{P'_R} = \mathbf{p_k + p_j} \tag{9}$$

II.3 Kaon-Nucleon Wave Functions Expansion

The kaon wave function $\varphi_\alpha(\mathbf{r})$, where α represents the collection of quantum numbers $(n_\alpha, J_\alpha, \ell_\alpha, m_\alpha)$, can be expanded in its radial, angular, and isotopic spin functions $\widehat{\mathbf{P}}_{\mathbf{T}_\alpha}$ parts as follows,

$$\varphi_\alpha(\mathbf{r}) = \sum_{\mathbf{m}_{\ell_\alpha}} (\ell_\alpha \mathbf{0} \ \mathbf{m}_{\ell\alpha}\mathbf{0} \mid \ell_\alpha \mathbf{m}_{\ell\alpha}) \varphi_{\mathbf{n}_\alpha \ell_\alpha \mathbf{m}_{\ell_\alpha}}(\mathbf{r}) \widehat{\mathbf{P}}_{\mathbf{T}\alpha} \tag{10}$$

It is to be noted that in this expansion there is no dependence on the spin function of the kaon. Similarly, expanding the nucleon wave function $\varphi_\gamma(\mathbf{r})$ in terms of the spin function $\chi^{1/2}_{m_{s\gamma}}$ and the isotopic spin function $\widehat{\mathbf{P}}_{\mathbf{T}\gamma}$ we have,

$$\varphi_\gamma(\mathbf{r}) = \sum_{\mathbf{m}_{\ell_\gamma} \mathbf{m}_{\mathbf{s}_\gamma}} (\ell_\gamma \ \mathbf{1/2} \ \mathbf{m}_{\ell_\gamma} \mathbf{m}_{\mathbf{s}_\gamma} \mid \mathbf{J}_\gamma \mathbf{m}_\gamma) \varphi_{\mathbf{n}_\gamma \ell_\gamma \mathbf{m}_{\ell_\gamma}(\mathbf{r}) \chi^{\mathbf{1/2}}_{\mathbf{m}_{\mathbf{s}\gamma}} \widehat{\mathbf{P}}_{\mathbf{T}\gamma}} \tag{11}$$

Consequently, the KN wave function can be written as follows,

$$\langle \varphi_\alpha(\mathbf{r_1}) \varphi_\gamma(\mathbf{r_2}) | = \sum_{\mathbf{m}_{\ell_\alpha} \mathbf{m}_{\ell_\gamma} \mathbf{m}_{\mathbf{s}_\gamma}} (\ell_\alpha \ \mathbf{0} \ \mathbf{m}_{\ell_\alpha} \mathbf{0} \mid \mathbf{J}_\alpha \mathbf{m}_{\ell\alpha})(\ell_\gamma \ \mathbf{1/2} \ \mathbf{m}_{\ell_\gamma} \mathbf{m}_{\mathbf{s}\gamma} | \mathbf{J}_\gamma \mathbf{m}_\gamma)$$

$$\langle \varphi_{n_\alpha \ell_\alpha m_{\ell_\alpha}}(\mathbf{r_1}) \varphi_{\mathbf{n}_\gamma \ell_\gamma \mathbf{m}_{\ell_\gamma}}(\mathbf{r_2}) \chi^{\mathbf{1/2}}_{\mathbf{m}_{\mathbf{s}_\gamma}}(\mathbf{2}) \widehat{\mathbf{P}}_{\mathbf{T_1}} \widehat{\mathbf{P}}_{\mathbf{T_2}} | \tag{12}$$

Then by using the relative and C.M. coordinate systems as defined by Eq.(8) we have,

$$\langle \varphi_{n_\alpha \ell_\alpha m_{\ell_\alpha}}(\mathbf{r_1}) \varphi_{\mathbf{n}_\gamma \ell_\gamma \mathbf{m}_{\ell_\gamma}}(\mathbf{r_2}) | = \sum_{\lambda\mu} (\ell_\alpha \ell_\gamma \mathbf{m}_{\ell_\alpha} \mathbf{m}_{\ell_\gamma} \mid \lambda\mu) \langle \varphi_{\mathbf{n}_\alpha \ell_\alpha \mathbf{n}_\gamma \ell_\gamma \lambda\mu}(\mathbf{r_1}, \mathbf{r_2}) | \tag{13}$$

We adopt an additional assumption that the expansion coefficients of the total KN wave function $\varphi(\mathbf{r_1}, \mathbf{r_2})$ when expanded in terms of the relative and C.M. coordinates, will be the generalized Talmi-Moshinsky-Smirnov (GTMS) brackets for particles with different masses [8],i.e.,

$$\langle \varphi_{n_\alpha \ell_\alpha n_\gamma \ell_\gamma \lambda\mu}(\mathbf{r_1}, \mathbf{r_2}) | = \sum_{\mathbf{n}\ell\mathbf{NL}} (\mathbf{n}_\alpha \ell_\alpha \mathbf{n}_\gamma \ell\gamma\lambda \mid \mathbf{NLn}\ell\lambda) \langle \varphi_{\mathbf{NLn}\ell\lambda\mu}(\mathbf{R\prime}, \mathbf{r\prime}) \ | \tag{14}$$

The important mathematical advantage we gain from the above assumption is that the motion of K^+N system is in the form of a harmonic oscillator which enables the separability of the wave function in its relative "$\mathbf{r\prime}$" and C.M."$\mathbf{R\prime}$" coordinates [9],

$$\langle \varphi_{NLn\ell\lambda\mu}(\mathbf{R}\prime,\mathbf{r}\prime) \mid = \sum_{\mathbf{Mm}} (\mathbf{L}\ell\mathbf{Mm} \mid \lambda\mu)\langle \varphi_{\mathbf{NLM}}(\mathbf{R}\prime) \mid \langle \varphi_{\mathbf{n}\ell\mathbf{m}}(\mathbf{r}\prime) \mid \tag{15}$$

Moreover, the spin and isotopic spin functions can be expanded as follows,

$$\langle \widehat{P_{T_\alpha}} \chi^{1/2}_{m_{s_\gamma}}(2) \widehat{P_{T_\gamma}} \mid = \sum_{sm_{s_\gamma} TM_T} (0\ 1/2\ 0\ m_{s_\gamma} \mid sm_{s_\gamma})(1/2\ 1/2\ T_\alpha T_\gamma \mid TM_T)\chi^{s}_{m_s}(2)\widehat{P_T}(1,2) \tag{16}$$

II.4 Kaon-Nucleon Potential

After somewhat tedious but straight forward mathematical manipulations we can write the K^+N interaction in the form,

$$\begin{aligned} V_{KN} &= \mathbf{V_a} - \frac{\mathbf{1}}{\mathbf{8M_2^2c^2}}(\mathbf{P\prime^2V_a} + \mathbf{V_aP\prime^2}) + (\frac{\mathbf{1}}{\mathbf{2M_2^2c^2}})(\frac{\mathbf{M_2}}{\mathbf{M_1+M_2}})^{\mathbf{2}}\mathbf{V_bP\prime_R^2} \\ &+ (\frac{\mathbf{1}}{\mathbf{4M_2c^2(M_1+M_2)}})[(\mathbf{P\prime_r.P\prime_R})\mathbf{V_a} + \mathbf{V_a}(\mathbf{P\prime_r.P\prime_R})] \\ &+ \frac{\mathbf{1}}{\mathbf{4M_2^2c^2}}[(\sigma_\mathbf{2}.\mathbf{P\prime_r})\mathbf{V_c}(\sigma_\mathbf{2}.\mathbf{P\prime_r}) - (\frac{\mathbf{M_2}}{\mathbf{M_1+M_2}})(\mathbf{V_c}[(\sigma_\mathbf{2}.\mathbf{P\prime_R})(\sigma_\mathbf{2}.\mathbf{P\prime_r})] \\ &+ [(\sigma_\mathbf{2}.\mathbf{P\prime_R})(\sigma_\mathbf{2}.\mathbf{P\prime_R}))]\mathbf{V_c})] \end{aligned} \tag{17}$$

where,

$\mathbf{V_a}(\mathbf{r}) = -\mathbf{J_s}(\mathbf{r}) + \mathbf{J_{v_1}}(\mathbf{r}) + \mathbf{J_{v_2}}(\mathbf{r}) + \mathbf{J_{v_3}}(\mathbf{r})$,

$\mathbf{V_b}(\mathbf{r}) = \mathbf{J_s}(\mathbf{r}) - \mathbf{J_{v_3}}(\mathbf{r})$, and

$\mathbf{V_c}(\mathbf{r}) = \mathbf{J_s}(\mathbf{r}) + \mathbf{J_{v_1}}(\mathbf{r}) + \mathbf{J_{v_2}}(\mathbf{r}) - \mathbf{J_{v_3}}(\mathbf{r})$

Then using the identities,

$(\sigma_\mathbf{2}.\mathbf{A})(\sigma_\mathbf{2}.\mathbf{B}) = (\mathbf{A}.\mathbf{B}) + \mathbf{i}(\sigma_\mathbf{2}[\mathbf{A}\times\mathbf{B}])$,

$(\sigma_\mathbf{2}.\mathbf{P})\mathbf{V_c}(\sigma_\mathbf{2}.\mathbf{P}) = \mathbf{V_cP^2} - \hbar^\mathbf{2}\frac{\mathbf{dV_c}}{\mathbf{dr}}\frac{\mathbf{d}}{\mathbf{dr}} + \frac{\mathbf{2}}{\mathbf{r}}\frac{\mathbf{dV_c}}{\mathbf{dr}}(\mathbf{s_2}.\ell)$, $\quad \mathbf{s_2} = \frac{\hbar}{\mathbf{2}}\sigma_\mathbf{2}$

$\mathbf{P_R} \equiv -\mathbf{i}\hbar\nabla_\mathbf{R}$, $\quad \mathbf{P} \equiv -\mathbf{i}\hbar\nabla_\mathbf{r}$, $\quad \mathbf{P}.\mathbf{P_R} = -\hbar^\mathbf{2}\frac{\mathbf{d}}{\mathbf{dR}}\frac{\mathbf{d}}{\mathbf{dr}}$

and substituting into Eq.(17) where from hereafter we omit the prime sign then we have,

$$
\begin{aligned}
V_{KN} &= \mathbf{V_a} - \frac{\mathbf{1}}{\mathbf{8M_2^2c^2}}(\mathbf{P^2V_a} + \mathbf{V_aP^2}) + \frac{\mathbf{1}}{\mathbf{2M_2^2c^2}}\left(\frac{\mathbf{M_2}}{\mathbf{M_1+M_2}}\right)^{\mathbf{2}}\mathbf{V_bP_R^2} \\
&+ \left(\frac{\mathbf{1}}{\mathbf{4M_2c^2(M_1+M_2)}}\right)[(\mathbf{P.P_R})\mathbf{V_a} + \mathbf{V_a}(\mathbf{P.P_R})] \\
&+ \frac{\mathbf{1}}{\mathbf{4M_2^2c^2}}[(\mathbf{V_c.P^2} - \mathbf{\hbar^2}\frac{\mathbf{dV_c}}{\mathbf{dr}}\frac{\mathbf{d}}{\mathbf{dr}} + \frac{\mathbf{2}}{\mathbf{r}}\frac{\mathbf{dV_c}}{\mathbf{dr}}(\mathbf{s_2}.\ell) \\
&- \left(\frac{\mathbf{M_2}}{\mathbf{M_1+M_2}}\right)[(\mathbf{V_c}(\mathbf{P_R.P}) + (\mathbf{P_R.P})\mathbf{V_c})] \qquad (18)
\end{aligned}
$$

II.5 Normalized Laugurre Polynomial and Yukawa-type Wave Functions

The normalized K^+N Laugurre functions $R_{nl}(r)$ can be written in the form,

$$R_{nl}(r) = [\frac{2(n!)}{\Gamma(n+l+3/2)}]^{1/2}\frac{1}{b^{3/2}}(\frac{r}{b})^l \exp(-(1/2)(\frac{r}{b})^2)L_n^{l+1/2}(\frac{r}{b})^2, \qquad (19)$$

In the K^+N system where inequal masses of the two particles, the size parameter "b" is defined in both relative and C.M. systems as follows [8],

$$b_r = \sqrt{\frac{\hbar(M_1+M_2)}{M_1M_2\omega}} \quad , \quad b_R = \sqrt{\frac{\hbar}{(M_1+M_2)\omega}}$$

In addition, to test the results, we use in our calculations for the meson functions a three Yukawa-type functions as follows,

(1)The associated Generalized Yukawa function [10] (GY),

$$J_i(r) = g_i^2\hbar c[\frac{\exp(-\mu_i r)}{r} - \frac{\exp(-\lambda r)}{r}(1+\frac{\lambda^2-\mu_i^2}{2\lambda}r)]; \mu_i = \frac{m_ic^2}{\hbar c} \qquad (20)$$

where, $m_i \equiv M_s, M_{v_1}, M_{v_2}, and \ \ M_{v_3}$ are the masses of the exchanged mesons

(2)The Single Particle Energy Dependent function [11] (SPED),

$$J_i(r) = c_i\hbar c[\frac{\exp(-A_ir)}{r} - \frac{\exp(-B_ir)}{r}], \qquad (21)$$

where, $\quad c_i = \frac{g_i^2}{4\pi}\frac{\Lambda_i^2}{\Lambda_i^2-m_i^2}, \quad A_i = \sqrt{m_i^2-(E_k-E_{k'})^2}, \quad B_i = \sqrt{\Lambda_i^2-(E_k-E_{k'})^2}$

(3) The Yukawa function of the form [12] (Y),

$$J_i(r) = (g_i^2/4\pi)\frac{\exp(-M_ir)}{r}, \quad i \equiv \sigma, \omega, \rho, and \ \ \sigma_0 \ \ mesons \qquad (22)$$

III. Parameterization

We follow in this work the parameterization given in [6b] for meson masses M_r, coupling constants g_r and their cut off parameters Λ_r as shown in Table.I for model A and Table.II for model B. Moreover, the kaon and nucleon masses M_1 and M_2 are taken respectively as follows,
kaon mass (M_1) = 495.82 MeV , nucleon mass (M_2) = 938.926 MeV and the separation energy parameter $\hbar\omega = 37.35$ MeV as suggested by the phenomenological formula, $\hbar\omega = 1.85 + \frac{35.5}{A^{1/3}}$

Table. I. *Vertex One-Body KN Meson Exchange Parameters (Model A)*

$Process[K^+N \rightarrow K^+N]$	J^P	I	$M_r[MeV/c^2]$	$g_r/\sqrt{4\pi}$	$\Lambda_r[GeV/c^2]$
$NN\rho$	1^-	1	769	0.917	1.4
$NN\omega$	1^-	0	782.6	4.472	1.5
$NN\sigma$	0^+	0	600	2.385	1.7

Table. II. *Vertex One-Body KN Meson Exchange Parameters (Model B)*

$Process[K^+N \rightarrow K^+N]$	$M_r[MeV/c^2]$	$g_r/\sqrt{4\pi}$	$\Lambda_r[GeV/c^2]$
$NN\rho$	769	0.917	1.4
$NN\omega$	782.6	2.750	1.5
$NN\sigma$	600	2.385	1.7
$NN\sigma_0$	1200	3.536	2

IV. Results, Discussion, and Conclusions

We have calculated the free K^+N radial potential form for three different meson Yukawa-type functions given by Eqs. (20), (21), and (22)using both of the parameters given in Table I and Table II respectively for both of model A, where the K^+N meson exchange process was constructed on the basis of three σ,ρ,ω exchanged mesons, and model B, where in addition to the above three mesons an additional repulsive σ_0 meson of more shorter range and heavier mass was added to account for the additional repulsion required by the data. Our results are shown in Figures 1,2, and 3. In Fig.1 it is plotted the radial dependence of the K^+N potential using the meson function (GY) given by Eq.(20) where the dotted line represents the potential obtained when using model A (three exchanged mesons), while the solid line represents the result obtained when using model B (four exchanged mesons). It is to be noted that this function gives a repulsive peak in the K^+N potential for both of the two models A and B at relative distance of the two particles r = 0.30 fm V(r=0.3 fm)= 170.66 MeV (model A), 175.58 MeV (model B). The maximum additional repulsion we gain as a result of the addition of the short range and heavier mass σ_0 meson (model B)in the K^+N interaction is 2.88 percent, at $r = 0.3fm$. It is important to denote that when using this GY meson function the repulsive additionally added σ_0 meson gives its exhaust (we suppose a repulsive ratio ≥ 1 percent) in the radial interval $r = 0.05 - 1.1fm$. In Fig.2 the radial dependence of the K^+N potential is plotted using the meson function (SPED) given by Eq.(21). We notice again a repulsive peak in the potential at r = 0.20 fm V(r=0.20 fm) = 408.13 MeV (model

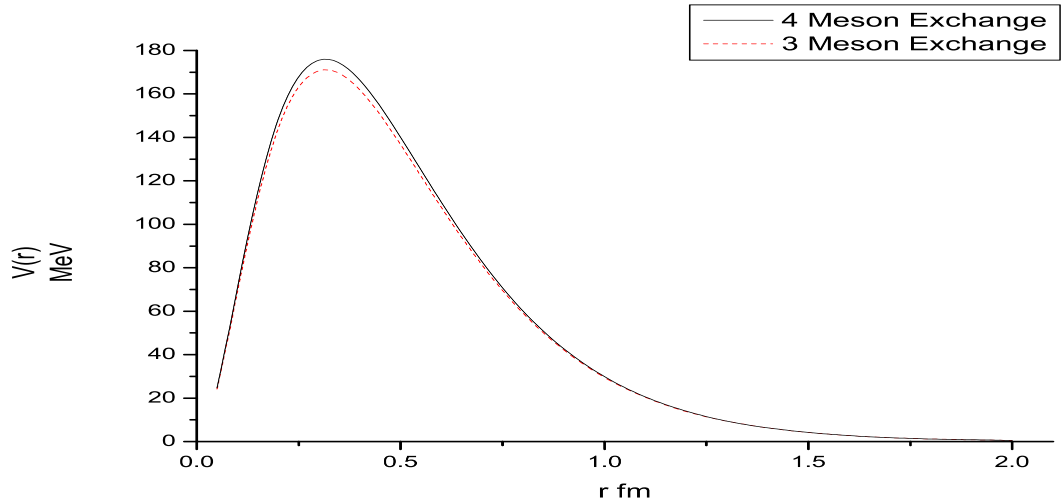

Figure 1: Kaon-Nucleon Radial Potential Using GY Meson Function

A), 421.03 MeV (model B)). Again we notice that the percentage maximum value of the additional repulsion due to the σ_0 meson found to be 3.53 percent at $r = 0.3fm$. It is to be noted that the repulsive values of the potential at the same relative distances are a rapidly varying function in comparison with their counterpart values obtained above when the GY meson function is used. We emphasize that the effective range of the SPED meson function for model B (four exchanged mesons) lies in the range $r = 0.05 - 1.3fm$. In Fig.3 where a pure simple Yukawa function (Y) as given by Eq.(22)is applied, it is noticed a pure repulsive nature of the potential along a radial range $r = 0.05 - 2.0fm$. The maximum increase in the repulsion ratio of the potential due to the addition of σ_0 meson in model B was found 2.48 percent at $r = 0.4fm$. We denote that due to the addition of the σ_0 repulsive meson this (Y) function gives its exhaust in the radial range ($r = 0.05 - 1.2fm$).We emphasize that the apparent different behavior of the three used meson functions (GY, SPED, and Y) on the absolute repulsive values, along the relative distance range, of the K^+N potential and, consequently, the difficulty to determine the physical eligibility of these functions, we believe that in definite realistic K^+A interactions using different concrete theoretical approaches, will enable us to differentiate decisively between these functions. In final, we summarize our conclusions as follows :

1) An analytic microscopic semi-relativistic K^+N potential was derived and suggested for use on the basis of One-Boson-Exchange-Potential model constructed on exchanging either three (one attractive and two repulsive) mesons or four (one attractive and three repulsive) mesons by adding the more shorter range and heavier σ_0 meson which accounts for the more repulsion required by the data.

2) The derived K^+N potential takes into account the correction effect due to the center of mass between the two different masses of the kaon and nucleon particles.

3) The K^+N derived potential have the two features required by the experimental data i.e. both the repulsive and short range characters.

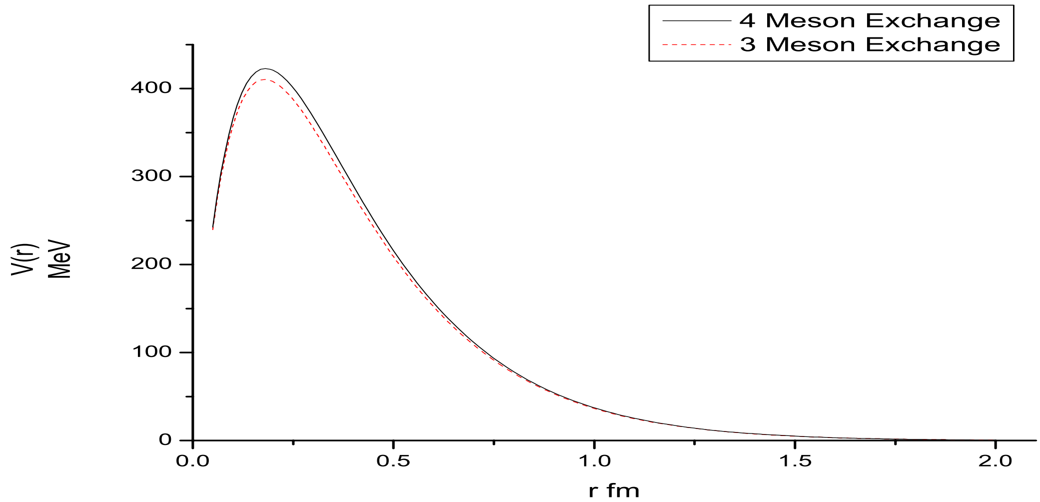

Figure 2: Kaon-Nucleon Radial Potential Using SPED Meson Function

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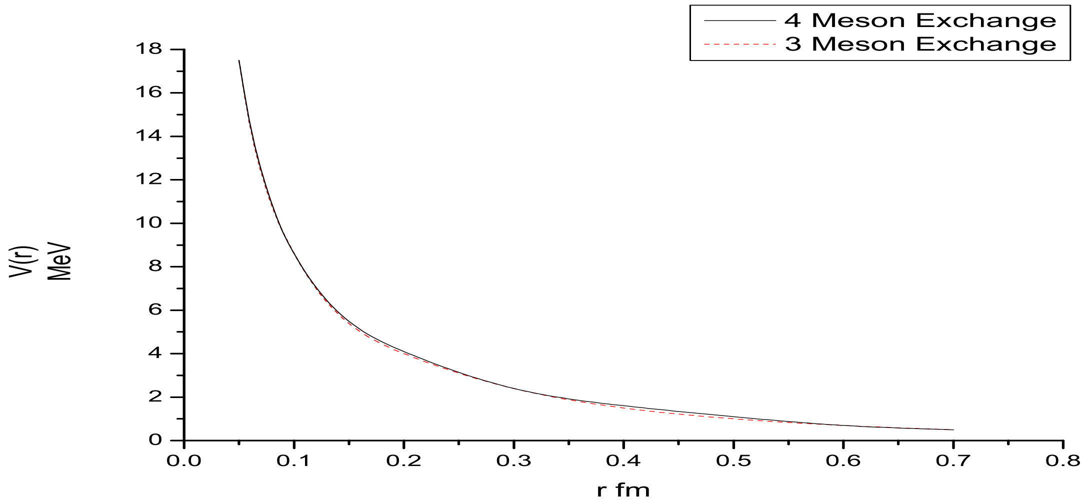

Figure 3: Kaon-Nuclon Radial Potential Using Y Meson Function

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