ANALYSIS OF ^{4}He - ^{4}He ELASTIC INTERACTIONS

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The $^4He - ^4He$ elastic scattering at P_L of 124.6 and 126 GeV/c is analyzed in the framework of the Glauber theory. The differential cross section evaluated using Mont Carlo method and by inclusion of the twelve quark bag admixture to the ground state of the ^4He nucleus in the calculations allows one to reproduce the experimental data quite well. It is shown that the admixture manifests itself in the $^4He - ^4He$ elastic scattering in all regions of the momentum transfer "t". At small t the effect can be found at the level of $\sim 10\%$. At large t it can be $\sim 30\%$.

Introduction

Understanding how nuclei react with each other is one of the fundamental goals of nuclear physics. One of the fundamental goals of nuclear physics is to know how nuclei behave in the reactions to devise knowledge about the structure of the nuclei and different aspects of the nuclear reactions.

The other interesting topic is the manifestation of the quarks in the reactions. In high energy interactions quarks and gluons receiving high transverse momentum are observed as jets of hadrons. At lower energies the jet production cross section becomes extraordinary small, and the jets have not been registered at experimental studies until now.

At the same time it is known that reactions with participation of the ^4He nucleus can not be described quite well within the framework of standard nuclear physics.

Dakhno and Nikolaev [1] have assumed and shown that 12% admixture of twelve quark bag configuration in the ground state wave function of the ^4He nucleus allows one to understand the irregularities of proton -^4He elastic scattering at high energies. Using this hypothesis, we have shown that it really gives an opportunity to describe $p - ^4He$ and $d - ^4He$ scattering in a wide energy range [2, 3]. In the present paper we continue our study, and consider ^{4}He - ^{4}He elastic scattering.

Experimental data on the ^{4}He - ^{4}He elastic scattering at the laboratory momentum of P_L of 124.6 and 126 GeV/c were presented in Refs. [4, 5, 6]. In the framework of the Glauber theory [7]. The key quantities of the theory are the characteristics of NN elastic scattering amplitude and the parametrization of the ground state wave function of the ^4He nucleus. The simplest Gaussian parametrization of the wave function was used, and the NN characteristics were considered as the fitting parameters.

It has been shown that the $12q$-bag admixture in the ground state of the ^4He nucleus manifests itself in the ^{4}He-^{4}He elastic scattering in all region of the momentum transfer.

As a result, all of these hid a big discrepancy between the experimental data and calculations with real parameters.

The inclusion of the twelve quark bag state of the ^4He nucleus in the calculation scheme and its manifestation to the state in the elastic scattering is considered in sec. 2. In the last sec. we summarize our results.

Calculation of $^4He - {^4He}$ elastic cross section

The Glauber amplitude of nucleus-nucleus scattering has the form [8, 9, 10]:

$$F_{AB}(\vec{q}) = \frac{ip}{2\pi} \int d^2b \; e^{i\vec{q}\cdot\vec{b}} \Gamma\left(\vec{b}\right), \tag{1}$$

$$\Gamma\left(\vec{b}\right) = \left\langle \psi_A^f \psi_B^f \left| \left[1 - \prod_{j=1}^{A} \prod_{k=1}^{B} \left(1 - \gamma(\vec{b} - \vec{s}_j + \vec{\tau}_k)\right) \right] \right| \psi_A^i \psi_B^i \right\rangle, \tag{2}$$

where $\vec{b}$ is the impact parameter, p is the momentum of the projectile nucleus, ψ_A^i, ψ_B^i and ψ_A^f, ψ_B^f are the initial and final states wave functions of the projectile and the target nucleus, respectively, γ is the NN elastic scattering amplitude in the impact parameter representation. In high energy physics γ is often parametrized as:

$$\gamma(\vec{b}) = \beta \; e^{-b^2/2B_{NN}}, \tag{3}$$

where $\beta = \sigma_{NN}^{tot}\left(1 - i\alpha_{NN}\right) / \left(4\pi B_{NN}\right)$, σ_{NN}^{tot} is the NN total cross section, B_{NN} is the slope parameter of the NN differential elastic cross section at zero momentum transfer, and α_{NN} is the ratio of the real to the imaginary parts of the NN elastic scattering amplitude at zero momentum transfer.

The elastic nucleus-nucleus differential cross section is determined by

$$\frac{d\sigma}{d\Omega} = \left|F_{AB}\right|^2. \tag{4}$$

To take into account all terms of the expansion of the product in Eq. (2), one can represent each term like that shown in Fig. 1, where the circles correspond to the interacting nuclei, the black and white points – to the nucleons, and the solid lines – to the interactions between nucleons.

Using the diagrams, one can calculate as much the diagrams of whatever type are. It is pure combinatorial problem which can be solved with the help of the graph theory. In this theory the diagrams of Fig. 1 are called bi-colored labeled graphs. These graphs can be represented with the help of an adjacency matrix. The adjacency matrix $\underline{\underline{D}} = [d_{ij}]$ of a labeled graph G is a matrix of order $A \times B$ in which $d_{ij} = 1$ if points i and j are adjacent (connected with a line) and $d_{ij} = 0$ in any other case. By another way, the graph can be represented by the set of crossing points of A horizontal and B vertical lines with dark circles in the places corresponding to the elements $d_{ij} = 1$. This representation is called the net graph representation. We will refer to the net graphs as the scattering diagrams. Each term in the expansion of Eq. (2) has the form

$$-\langle \psi_A^f \psi_B^f | \prod_{(j,k)} \left(-\gamma(\vec{b} - \vec{s}_j + \vec{\tau}_k)\right) | \psi_A^i \psi_B^i \rangle, \tag{5}$$

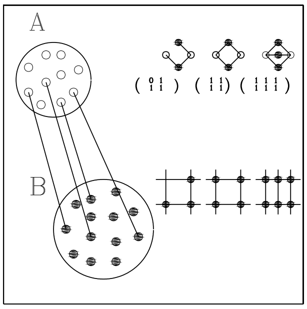

Figure 1: Graphical representation of the multiple scattering terms

$$(j,k) \in M \subset \{I_A\} \otimes \{I_B\}$$
$$\{I_A\} = (1,2,3,...,A), \qquad \{I_B\} = (1,2,3,...,B).$$

Because γ can be represented by a graph G or by the corresponding matrix $\underline{\underline{D}}$ we will consider the term as a graph function $\mathfrak{g}(\underline{\underline{D}})$. The scattering amplitude Γ_{AB} now can be re-written as

$$\Gamma(\vec{b}) = \sum_{\mu} H_{\mu} \cdot \mathfrak{g}\left(\underline{\underline{D_{\mu}}}\right), \tag{6}$$

where summation runs on the set of all nonisomorphic graphs. From graph theory we have that the combinatorial coefficient at the function of graph G_μ with $l-$components, $k_1$belonging to one class of isomorphism, k_2 to another class, etc., $(l = k_1 + k_2 + \cdots + k_j)$ is equal to

$$\begin{aligned} H_{\mu} &= \frac{A!}{(m_1!)^{k_1}(m_2!)^{k_2}\cdots(m_j!)^{k_j}(A-\sum_{i=1}^{j} m_i k_i)!} \times \\ & \frac{B!}{(n_1!)^{k_1}(n_2!)^{k_2}\cdots(n_j!)^{k_j}(B-\sum_{i=1}^{j} n_i k_i)!}\prod_{i=1}^{j}\left(\frac{m_i!n_i!}{\mathrm{S}(G_i)}\right), \end{aligned} \tag{7}$$

where m_i and n_i are the numbers of the points of the sets A and B, respectively, in the component belonging to the i-th class of isomorphism, and $\mathrm{S}(G_i)$ is the number of symmetries of this component.

In the case of the elastic $^4He - {}^4He$ scattering all the $\underline{\underline{D}}$'s matrices will consider At $A = 4$ and $B = 4$ the amplitude F_{AB} given by Eq. (1) will be,

$$F_{44}(\vec{q}) = \frac{ip}{2\pi}\int d^2b\; e^{i\vec{q}\cdot\vec{b}}\Gamma\left(\vec{b}\right), \tag{8}$$

$$\Gamma\left(\vec{b}\right) = \left\langle \psi_d^i \psi_{He}^i \left| \left[1 - \prod_{j=1}^{4}\prod_{k=1}^{4}\left(1-\gamma(\vec{b}-\vec{s}_j+\vec{\tau}_k)\right)\right] \right| \psi_d^i \psi_{He}^i \right\rangle. \tag{9}$$

The modules of ψ_{He} square was taken as [1].

$$|\psi(\vec{r}_1,\ldots,\vec{r}_4)|^2 = (2\pi)^3\rho_c\delta\left(\sum_{i=1}^{4}\vec{r}_i\right)\prod_{i=1}^{4}\varphi(\vec{r}_i). \tag{10}$$

We use the two following parameterizations of $\varphi(\vec{r})$ [2],

$$(I) \qquad \varphi(\vec{r}) = exp[-\vec{r}^2/R_1^2],$$
$$(II) \qquad \varphi(\vec{r}) = exp[-\vec{r}^2/R_1^2] + D_1 exp[-\vec{r}^2/R_2^2] - (1 + D_1 - D_2^2) exp[-\vec{r}^2/R_3^2].$$

where the parameters are given in Table 1.

	R_1^2 $(GeV/c)^{-2}$	R_2^2 $(GeV/c)^{-2}$	R_3^2 $(GeV/c)^{-2}$	D_1	D_2
I	51.01				
II	62.06	19.0	10.13	3.79	0.31

Table 1: Values of used parameters (from [1])

We will use a general form for the function φ, that is,

$$\varphi(\vec{r}) = \sum_{i=1}^{N} C_i e^{r^2/R_i^2}. \qquad (11)$$

By the same method performed in Ref. [3] we will follow it.

The twelve quark bag admixture in the ^{4}He-Nucleus cross sections

The direct measurement of the elastic nucleus-nucleus cross section at high energies (larger than few GeV per nucleon) is a complicated problem due to the small scattering angle and the insufficient energy resolution for the incident and the scattering particles. For this reason the experimental data is rather limited. The elastic and quasi-elastic (with target nucleus destruction) ^{4}He-nucleus scattering at $P_L = 17.9$ GeV/c have been measured at JINR, Dubna. Elastic ^{4}He-^{4}He-scattering at $\sqrt{s} \simeq 126$ GeV have been studied at CERN in three experiments R210 $(0.05 < |t| < 0.8 GeV^2)$, R418 $(0.2 < |t| < 0.8 GeV^2)$ and R807$(0.05 < |t| < 0.19 GeV^2)$. The experiments were performed at the same four momentum transfers ranges, however there was a disagreement between the data shown in figure (2). In order to estimate the multi-quark bag effects we have used the same assumptions as in the previous Chapters 1 and 2 [2, 3]. We write the elastic scattering amplitude in the form:

$$F_{\alpha\alpha} = (1 - W_{12q})^2 F_{4N,4N} + 2W_{12q}(1 - W_{12q}) F_{4N,12q} F_{4N,12q} + W_{12q}^2 F_{12q,12q}, \qquad (12)$$

where $F_{4N,4N}$ represents the amplitude when both α-particle are in 12q-bag states. The last term in equation (12) gives bag-bag scattering amplitude.

The first term was calculated as described in Ref. [3] with the summation of full Glauber series using one-Gaussian parametrization of ^{4}He one-particle density. In the calculations, 316 diagrams have been considered. Evaluations of the second term has been given in chapter 1 where it is considered as hadron-^{4}He amplitude albeit using the $N - 12q$ parametrization. For the third term we used simple Gaussian parametrization. The calculations presented in figure (2) shows a low sensitivity of the differential cross-section to the third term. It is quite understandable because it has the small coefficient, $W_{12q} \simeq 0.01$.

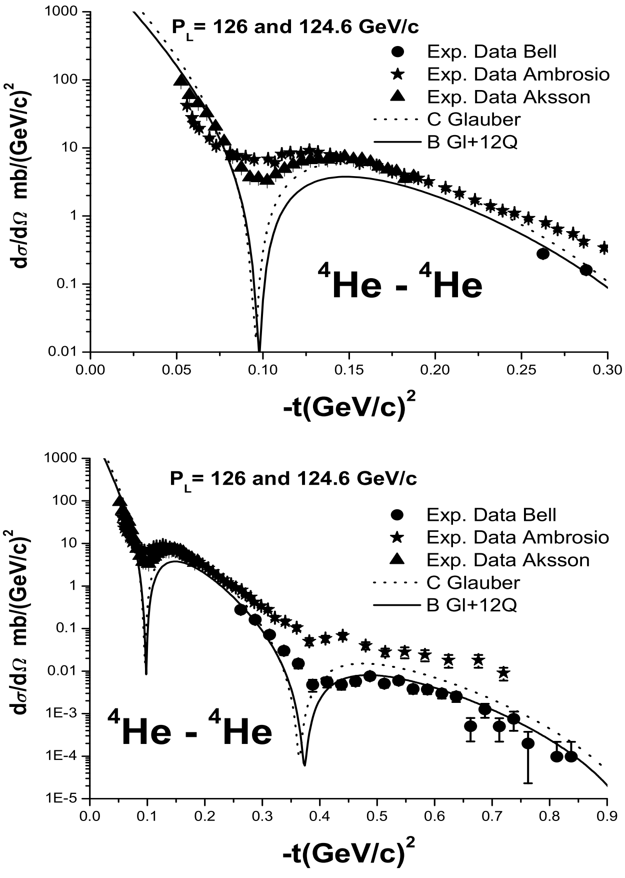

Figure 2: ^{4}**He-**^{4}**He differential cross-sections. The experimental data of references [4, 5, 6] compared with Glauber calculations with and without 12q-bags.**

As it can be seen from the figure (2), the Glauber model calculations overestimate the differential cross-sections in the region of the third maximum. Inclusion of $12q$-bag state puts the calculations in a satisfactory agreement with the experimental data. This is due to the second term of the amplitude. Thus the peculiarity of p-^{4}He scattering reflects in ^{4}He-^{4}He elastic scattering at $|t| < 0.8 GeV^2$. The properties of bag-bag interactions can not be extracted from the data.

It should be noted that a theoretical description of the data has been considered in a paper by Alberi et al. [11]. An unusual large inelastic screening correction was found. In contrast, we do not need any of such corrections in our calculation.

We assumed 10% admixture of 12q-bag state to the ^{4}He ground state wave function. One-particle nuclear density of heavy nuclei was chosen in the Woods-Saxon form. Parameters of NN-interaction were taken from the particle data group tables [12, 13]. As it is seen, the pure Glauber approximation does not allow one to describe satisfactory the data. Inclusion of the 12q-bag in ^{4}He nucleus brings the calculation to an agreement with the experimental data. The influence of bag-bag interaction is small, and can not be clearly shown in the figure.

Summing up we conclude that the hypothesis on 12q-bag admixture to the ^{4}He ground state allows one to overcome the old problems of ^{4}He-nucleus interactions.

Conclusion

The differential ^{4}He-^{4}He cross section has been estimated by using the graph method of Ref. [3] and the Monte Carlo method with the inclusion of the quasi-elastic scatting. It is shown that the $12q$-bag admixture with the ground state of the ^4He nucleus manifests itself in the ^{4}He-^{4}He elastic and quasi-elastic scattering in all region of the momentum transfer. At small t the effect can be about 10 %. At large t it reaches the value of about $\sim$ 30 %.

Acknowledgement:

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