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CONTINUOUS STOCHASTIC NEUTRON TRANSPORT FOR PURE-TRIPLET SCATTERING IN SEMI-INFINITE MEDIA USING GAUSSIAN STATISTICS

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Abstract

The stationary solution of the one-speed neutron transport equation in a semi-infinite stochastic medium with pure-triplet scattering is considered. The cross section function of the medium is assumed to be a continuous random function of position, with fluctuations about the mean taken as Gaussian distributed. The joint probability distribution function of these Gaussian random variables is used to calculate the ensemble-averaged solution for an arbitrary correlation function. The problem is solved at first in the deterministic case, then the solution is averaged using Gaussian joint probability distribution function. Numerical results are given for the radiant neutron energy and net neutron flux.

Keywords: *Neutron transport, Stochastic media, Gaussian statistics, Pure-triplet scattering*

INTRODUCTION

In the last two decades, there has been some interest in formulating linear kinetic theory and particle transport descriptions in a random medium. By a random we mean that the properties of the background material of the medium, with which the particles interact, are only known in a statistical sense. In our terminology, this would be called "stochastic transport theory". We use the terms stochastic and nonstochastic (deterministic) here in a special sense. That is the particle transport, involving the interaction of individual particles with a background material, is itself a stochastic process but this is not the stochasticity that we mean when we use the terms "stochastic transport" and "nonstochastic transport." Our use of the word, nonstochastic means that the properties of the medium, as functions of position and time, are either specified or can be computed in a deterministic fashion.

In the traditional statistical description of the transport of particles in a stochastic medium, the stochasticity arises on the atomic scale due to the random location of scattering centers. Assuming that interaction ranges are small in comparison with the distance between collisions and that collision duration is small in comparison with the time between collisions. On averaging the particle motion over these lengths and time scales, the celebrated

Boltzmann equation is obtained. The latter is a deterministic equation for the expected (or mean) density of particles in a unit phase space volume. Fluctuations about this mean density occur on lengths and time scales much shorter than the mean free paths and mean times between collisions. Their description requires a higher order treatment.

In most applications involving linear transport processes, however, the magnitude of fluctuations is generally small enough so that a description based on the expected number suffices, e.g., as in the traditional transport equation for neutrons, photons, charged particles, etc. [1]. Thus one deals with a purely deterministic equation characterized by microscopic parameters such as cross sections and macroscopic parameters such as the environmental density and temperature. The flow of neutral particles through the medium, which interact with a background material, but not with themselves, is described in some generality by this transport equation. While it is algebraically complex, it has a very simple physical content. It is simply the mathematical statement of particles conservation in phase space. Much of the fundamental understanding of physics contained in this equation, and the development of elegant mathematical methods to describe this physics, was pioneered by Chandrasekhar [1], Case and Zweifel [2] and Pomraning [3].

The stochastic transport class of problems arises when the environmental properties of the background material of the medium are random functions of position and time. The scientific texts treating the stochastic transport can essentially be divided into two major classes, in accordance with the definition of the randomness of background material properties. The first class deals with two-phase random media (discrete stochastic media). Models of this kind were elaborated in detail by many authors (cf. e.g. [4], [5] and the references cited therein). In this case the most extensive results have been obtained for the so-called Markovian mixtures.

The second class of stochastic media is related to the theory of fluctuations (continuous stochastic media). The advection and dispersion of a passive scalar (concentration, temperature, etc.) in a turbulent fluid is a classic example of this phenomenon, where the fluid velocity is assumed to be a random function with known statistical properties [6]. In boiling water reactors, coolant density and temperature fluctuations can be considered to introduce randomness in the flux by causing the neutron mean free path to fluctuate. Similarly, one can speculate that the transport of photons in stellar atmospheres is random on mean free path length scales as a result of temperature fluctuations. An application that motivated this work is the transport of neutral atoms in tokamak plasmas [7]. Such plasmas are known to be turbulent, and to a good approximation, the plasma density variations are well represented by a Gaussian stochastic process. Also, the model of Gaussian fluctuations has been used to investigate the effect on the dose due to a charged-particle beam of random fluctuations in the material density [8].

In models belonging to the second class one defines, as a rule, the properties of the medium as a Gaussian random function. Here, we can quote some papers by Prinja as the leading author [7]-[10]. In former papers [11]-[13], we had also dealt with Gaussian random media. The statistical moments of the stochastic quantities have been calculated , with the cross section function of the medium defined as a Gaussian random function.

In this paper we extend the previous Gaussian model [11]-[13], to the case of neutron transport in a semi-infinite medium with pure-triplet scattering instead of the linear anisotropic scattering. The advantage of this model is that it is easy to characterize, requiring only the mean, variance, and two-point correlation function to completely specify the distribution of random field. In section 2, we have obtained a closed form for the solution of the neutron transport equation in a semi infinite medium (deterministic solution). The reflectivity at the boundary as well as the radiant neutron energy and the net neutron flux are calculated. The medium is treated with a given value of angular-dependent externally-incident flux of neutrons on a specular-reflecting boundary. Section 3 represents the statistical analysis of the problem. In this section we clarify the stochasticity of the cross section, $\sigma(z)$, its mean, variance, and autocorrelation function. Gaussian probability is used to obtain the average radiant neutron energy and the net neutron flux. Section 4 presents some numerical results of our calculations for different values of specular reflectivity and scattering albedo.

ANALYSIS

Consider the monoenergetic time-dependent, one speed neutron transport equation with anisotropic scattering as [20], [21]

$$\left[\mu\frac{\partial}{\partial z}+\sigma(z)\right]N(z,\mu)=\frac{\sigma_s(z)}{2}\int_{-1}^{1}P(\mu,\mu')N(z,\mu')d\mu' \quad \textbf{(1)}$$

$$0\le z<\infty,\quad -1\le\mu\le 1$$

where

- $N(z, \mu)$ is the neutron density distribution function with geometrical space variable z, and the angular variable μ (the direction cosine of the transported neutrons).
- $\sigma(z)$ is the total cross-section (which is the total probability of a neutron interaction), $\sigma(z) = \sigma_a(z) + \sigma_s(z)$.
- $\sigma_a(z)$ is the absorption cross-section (which is the probability that a neutron at position z and time t will be absorbed).
- $\sigma_s(z)$ is the scattering cross-section (which is the probability that a neutron at position z and time t will have a collision and scatter).
- $P(\mu,\mu')$ is the anisotropic scattering phase function which can be expanded by terms of Legendre polynomials as [16]

$$P(\mu,\mu')=\sum_{n=0}^{\infty}a_nP_n(\mu)P_n(\mu')$$

$$=1+a_1\mu\mu'+a_2P_2(\mu)P_2(\mu')+a_3P_3(\mu)P_3(\mu')+\cdots \quad \textbf{(2)}$$

with $P_n(\mu)$ is the n^{th} Legendre polynomial function.

Here a_n can be called the anisotropy scattering coefficients with $a_0 = 1$. The first term of this expansion is called isotropic scattering ($a_1 = a_2 = a_3 = 0$). The probability of particle scattering is equal for all directions in isotropic scattering. The second term in this expansion corresponds to the linearly anisotropic scattering ($a_1 \neq 0$, $a_2 = a_3 = 0$). The third term corresponds to the quadratic scattering (Rayleigh scattering $a_1 = 0$, $a_2 = 0.5$, $a_3 = 0$). The fourth term corresponds to the triplet scattering ($a_1 = a_2 = 0$, $a_3 \neq 0$). Here, the scattering of neutrons from nucleus is considered as pure-triplet [17].

It is convenient to write Eq.(1) in terms of the optical depth space variable [4]

$$\tau(z)=\int_0^z\sigma(z)dz \quad \textbf{(3)}$$

to become

$$\left(\mu\frac{\partial}{\partial\tau}+1\right)I(\tau,\mu)=\frac{\omega}{2}\int_{-1}^{1}P(\mu,\mu')I(\tau,\mu')d\mu' \quad \textbf{(4)}$$

where

$$I(\tau,\mu)\equiv N(z,\mu) \quad \textbf{(5)}$$

and the single scattering albedo, which is independent of z (homogeneous medium), is given by

$$\omega=\sigma_s/\sigma \quad \textbf{(6)}$$

Equation (4) is assumed to subject to the specular-reflecting boundary conditions

$$I(0,\mu)=\Gamma(\mu)+\rho^{s}I(0,-\mu)\ ,\tag{7}$$

$$\lim_{\tau\to\infty}I(\tau,-\mu)=0\tag{8}$$

where $\Gamma(\mu)$ is the angular-dependent externally-incident flux, and ρ^s is the specular reflectivity of this boundary.

For pure-triplet scattering, we get Eq.(4) as

$$\left(\mu\frac{\partial}{\partial\tau}+1\right)I(\tau,\mu)=\frac{\omega}{2}\int_{-1}^{1}\left[1+a_3P_3(\mu)P_3(\mu')\right]I(\tau,\mu')d\mu'\tag{9}$$

The transport equation of the type given by Eqs.(9) admits separable exponential solutions of the form

$$I(\tau,\mu)=A\Psi(\mu,\nu)\exp(-\nu\tau)\tag{10}$$

where $\Psi(\mu,\nu)$ is a normalized function and A is the normalization constant to be determined.

Using of Eq.(10) in Eq.(9) yields

$$(1-\nu\mu)\Psi(\mu,\nu)=\frac{\omega}{2}\left[h_0(\nu)+a_3P_3(\mu)h_3(\nu)\right]\tag{11}$$

with

$$h_i(\nu)=\int_{-1}^{1}P_i(\mu)\Psi(\mu,\nu)d\mu\tag{12}$$

Integrating Eq.(11) over $\mu\in[-1,1]$ gives

$$(1-\omega)h_0(\nu)-\nu h_1(\nu)=0\tag{13}$$

Multiplying Eq.(11) by μ and μ^2 and integrate over $\mu\in[-1,1]$, one gets, respectively

$$h_1(\nu)-\frac{\nu}{3}\left[h_0(\nu)+2h_2(\nu)\right]=0\tag{14}$$

$$\frac{1}{3}\left[h_0(\nu)+2h_2(\nu)\right]-\frac{\nu}{5}\left[3h_1(\nu)+2h_3(\nu)\right]=\frac{\omega}{3}h_0(\nu)\tag{15}$$

Dividing Eq.(11) by $(1-\nu\mu)$ and integrate over $\mu\in[-1,1]$ we get

$$\left[1-\frac{\omega}{\nu}q_0\left(\frac{1}{\nu}\right)\right]h_0(\nu)-\frac{a_3\omega}{\nu}q_3\left(\frac{1}{\nu}\right)h_3(\nu)=0\tag{16}$$

where we have defined the n<u>th</u> order Legendre function of second kind, $q_n(y)$, as

$$q_n(y)=\frac{1}{2}\int_{-1}^{1}\frac{P_n(\mu)}{y-\mu}d\mu\tag{17}$$

with

$$q_0\left(\frac{b}{\nu}\right)=\frac{1}{2}\ln\left(\frac{b+\nu}{b-\nu}\right)\tag{18.a}$$

$$q_3\left(\frac{b}{\nu}\right)=\frac{b}{4\nu^3}\left(5b^2-3\nu^2\right)\ln\left(\frac{b+\nu}{b-\nu}\right)-\frac{5b^2}{2\nu^2}+\frac{2}{3}\tag{18.b}$$

Eqs.(13)-(16) constitute four linear homogeneous algebraic equations for the four unknowns h_0, h_1, h_2 and h_3. The corresponding vanishing of the coefficient determinant gives the transcendental equation (characteristic equation) satisfied by ν, as

$$\begin{vmatrix} 1-\omega & -\nu & 0 & 0 \\ -\dfrac{\nu}{3} & 1 & -\dfrac{2\nu}{3} & 0 \\ \dfrac{1-\omega}{3} & -\dfrac{3\nu}{5} & \dfrac{2}{3} & -\dfrac{2\nu}{5} \\ 1-\dfrac{\omega}{\nu}q_0 & 0 & 0 & \dfrac{-a_3\omega}{\nu}q_3 \end{vmatrix} = 0 \qquad \textbf{(19)}$$

Now for normalized $\Psi(\mu,\nu)$ [i.e. $h_0(\nu) = 1$], we have

$$h_1(\nu) = \frac{1}{\nu}(1-\omega) \qquad \textbf{(20)}$$

$$h_2(\nu) = \frac{1}{2}\left[1-\frac{3}{\nu^2}(1-\omega)\right] \qquad \textbf{(21)}$$

and

$$h_3(\nu) = \frac{5}{6\nu} - \left(\frac{5}{2\nu^3}+\frac{2}{3\nu}\right)(1-\omega) \qquad \textbf{(22)}$$

Therefore, from Eq.(11) we obtain

$$\Psi(\mu,\nu) = \frac{\omega/2}{(1-\nu\mu)}\left\{1+a_3P_3(\mu)\left[\frac{5}{6\nu}-(1-\omega)\left(\frac{5}{2\nu^3}+\frac{2}{3\nu}\right)\right]\right\} \qquad \textbf{(23)}$$

and hence we obtain the solution in the form

$$I(\tau,\mu) = \frac{\omega A}{2(1-\nu\mu)}\left\{1+a_3P_3(\mu)\left[\frac{5}{6\nu}-(1-\omega)\left(\frac{5}{2\nu^3}+\frac{2}{3\nu}\right)\right]\right\}\exp(-\nu\tau) \qquad \textbf{(24)}$$

The constant A can be determined by introducing a weight function $W(\mu)$ in order to force the boundary condition Eq.(7) to be fulfilled, as

$$\int_0^1 d\mu W(\mu)\left[I(0,\mu)-\Gamma(\mu)-\rho^s I(0,-\mu)\right] = 0 \qquad \textbf{(25)}$$

this can give

$$A = \left(\frac{2}{\omega}\right)\frac{I_0}{I_+-\rho^s I_-} \qquad \textbf{(26)}$$

where

$$I_0 = \int_0^1 W(\mu)\Gamma(\mu)d\mu \quad \text{and} \quad I_\pm = \int_0^1 W(\mu)J(\pm\mu)d\mu \qquad \textbf{(27)}$$

with

$$J(\pm\mu) = \frac{1}{(1\mp\nu\mu)}\left\{1+a_3P_3(\mu)\left[\frac{5}{6\nu}-(1-\omega)\left(\frac{5}{2\nu^3}+\frac{2}{3\nu}\right)\right]\right\} \qquad \textbf{(29)}$$

The solution is, then given by

$$I(\tau,\mu) = \frac{I_0}{I_+-\rho^s I_-}J(\mu)\exp(-\nu\tau) \qquad \textbf{(30)}$$

Equation (30) represents the explicit form of the deterministic analytical solution for the problem under consideration. Now, we can calculate the reflectivity at the boundary of the semi-infinite medium as

$$R = \int_0^1 \mu I(0,-\mu)d\mu, \qquad \textbf{(31)}$$

this gives

$$R = \frac{I_0 I_r}{I_+ - \rho^s I_-} \tag{32}$$

where

$$I_r = \int_0^1 \mu J(-\mu)\, d\mu \tag{33}$$

Further, we could calculate the radiant energy and the net flux of the transported neutrons, respectively, as

$$E(\tau) = \int_{-1}^{1} I(\tau,\mu)\, d\mu, \tag{34}$$

$$F(\tau) = \int_{-1}^{1} \mu I(\tau,\mu)\, d\mu \tag{35}$$

Using Eq.(30) in Eqs.(34) and (35), we get the deterministic values of $E(\tau)$ and $F(\tau)$ as

$$E(\tau) = \frac{I_0 I_e}{I_+ - \rho^s I_-} \exp(-\nu\tau), \tag{36}$$

$$F(\tau) = \frac{I_0 I_f}{I_+ - \rho^s I_-} \exp(-\nu\tau) \tag{37}$$

where

$$I_e = \int_{-1}^{1} J(\mu)\, d\mu, \tag{38}$$

$$I_f = \int_{-1}^{1} \mu J(\mu)\, d\mu \tag{39}$$

STATISTICAL ANALYSIS

In the usual (nonstochastic) application of the transport equation, Eq.(1), the total cross section, $\sigma(z)$, and the scattering kernel, $\sigma_s(z)$, are known (deterministic) prescribed functions. The goal is simply to solve this equation with respect to the boundary conditions given by Eq.(7) for $N(z,\mu)$. In the stochastic setting $\sigma(z)$ and $\sigma_s(z)$ are only known in some probabilistic sense. That is, at each space point z there is some probability that each of these two quantities will assume certain values. Accordingly, we consider $\sigma(z)$ and $\sigma_s(z)$ to be random functions, and then $N(z,\mu)$ is also random. On transforming Eq.(1) from the geometrical space to the optical space, Eq.(4), the stochasticity of the problem is absorbed in the optical variable τ. It is also assumed here that the cross section is a random function of position such that the single scattering albedo $\omega = \sigma_s/\sigma$ is not random.

On treating the random function $\sigma(z)$, firstly, we assume that it is a statistically homogeneous random function. This means that if we replace all values of z_i by $z + z_i$, (i = 1,2, … , m), the average values of the product $\sigma_1(z_1)\ \sigma_2(z_2)\ \sigma_3(z_3)\ ...\ \sigma_m(z_m)$, (m = 2,3, ….) do not depend on the geometrical depth space variable z. Secondly, we exemplify $\sigma(z)$ by a Gaussian random function with a constant mean value $\bar{\sigma} = \langle \sigma(\mathrm{z}) \rangle$ and a constant variance η_σ^2. From the probabilistic point of view, any Gaussian random function is defined completely if its mean, $\bar{\sigma}$, and its autocorrelation function, $W_\sigma(z_1,z_2)$, are defined. We define the autocorrelation of the random function $\sigma(z)$ as

$$W_\sigma(z_1,z_2) = \langle [\sigma_1(z_1)-\bar{\sigma}][\sigma_2(z_2)-\bar{\sigma}] \rangle \quad \textbf{(40)}$$

where z_1 and z_2 are arbitrary positions. For homogeneous statistics, $W_\sigma(z_1,z_2)$ depend on $|z_1 - z_2|$. It can be expressed as

$$W_\sigma(z_1,z_2) = W_\sigma(|z_1-z_2|) = \eta_\sigma^2 B(|z_1-z_2|) \quad \textbf{(41)}$$

where the variance η_σ^2 is given by

$$\eta_\sigma^2 = \langle [\sigma(z)-\bar{\sigma}]^2 \rangle > 0 \quad \textbf{(42)}$$

We assume that the function $W_\sigma(z_1,z_2)$ is positive for real values of z. Moreover, $W_\sigma(0)=\eta_\sigma^2$ and $W_\sigma(z) \to 0$ as $z \to \infty$. The shape function, B, describes the range over which the parameters fluctuations are correlated. It is typically characterized by correlation length ℓ such that $B \approx 0$ for $|z_1 - z_2| >> \ell$ and $B(0) = 1$. The form of B depends on the specific application, but in many cases, it is modeled by simple exponentials. However, the shape function, $B(|z_1-z_2|)$ is a deterministic function and must be given in advance. Important and common employed models are the exponentials [8], [11]

$$B(|z|) = \exp\left(-\frac{|z|}{\ell}\right) \quad \textbf{(43)}$$

or

$$B(|z|) = \exp\left(-\frac{z^2}{\ell^2}\right) \quad \textbf{(44)}$$

If σ_1 and σ_2 are two Gaussian random functions, then $\sigma_1+\sigma_2$ is also Gaussian random function. Generalizing this statement, we may say that, if $\sigma(z)$ is a Gaussian random function, then the optical depth space variable τ is also Gaussian. This fact follows from the linearity of the formula (3). The mean value of τ is

$$\bar{\tau} = \langle \tau \rangle = \int_0^z dz' \langle \sigma(z') \rangle = \bar{\sigma} z \quad \textbf{(45)}$$

The variance η^2 of τ depends on the form of the function $W(|z|)$ and can be written as

$$\eta^2 = \langle (\tau-\bar{\tau})^2 \rangle = \int_0^z dz_1 \int_0^z dz_2 \langle [\sigma(z_1)-\bar{\sigma}][\sigma(z_2)-\bar{\sigma}] \rangle = \int_0^z dz_1 \int_0^z dz_2 W_\sigma(|z_1-z_2|) \quad \textbf{(46)}$$

We refrain from calculating the parameter η and take simply both $\tau > 0$ and $\eta > 0$ as given constants. We require that

$$0 < \eta_\sigma << \bar{\tau} \quad \textbf{(47)}$$

However, the randomness of the function $\sigma(z)$ is defined by three parameters: $\bar{\sigma}$, η_σ and the correlation length ℓ. If $\eta_\sigma \to 0^+$, our stochastic problem goes over into the deterministic one. The mean value $\bar{\tau}$ of the optical variable τ is dimensionless quantity proportional to the factual length z, Eq.(45). It is suitable to introduce a positive dimensionless parameter

$$\beta = \eta_\sigma \sqrt{\frac{\ell}{\bar{\sigma}}} \quad \textbf{(48)}$$

The constants $\bar{\sigma}$, η_σ and ℓ (and then also the parameter β) are statistical characteristics of the random media. The value of β may be more or less arbitrary, serving for the characterization of the random medium under consideration. To estimate the value of the quantity η according to formula (46), we have to choose the shape of the autocorrelation function $W(|z|)$. Let us take $W(|z|)$ in the exponential form [8], [11] (see formulae (43))

$$W(|z|) = \eta_\sigma^2 \exp\left(-\frac{|z|}{\ell}\right) \quad \textbf{(49)}$$

Using Eq. (49) in Eq.(46), assuming that $\ell << z$, we get

$$\eta^2 \approx 2\eta_\sigma^2 \ell z \approx 2\beta^2 \bar{\sigma} z \quad \textbf{(50)}$$

If $W(|z|)$ was chosen in a form differing from the simple exponential, or if the correlation length ℓ was defined with a factor different from unity, formula (50) would still be valid, although with a factor different from 2. For the sake of simplicity, we will take formula (50) as approximately correct even in the case of small values of z.

THE AVERRAGE SOLUTION

In the previous treatment we have assumed that τ is a Gaussian real random variable and $\bar{\tau} = \langle \tau \rangle \geq 0$. In defining τ as a Gaussian random variable, we have to assume that values of τ may span the whole real axis, including the negative values. We have to require the existence of the variance $\eta^2 > 0$. The Gaussian probability density of τ is then

$$P_G(\tau, \eta^2) = \frac{1}{\sqrt{2\pi\eta^2}} \exp\left(-\frac{(\tau - \bar{\tau})^2}{2\eta^2}\right) \quad \textbf{(51)}$$

It is easy to verify that the averaged value of $\exp[-k(\tau - \bar{\tau})]$ with a constant k, is

$$Z_G(k, \eta^2) = \langle \exp[-k(\tau - \bar{\tau})] \rangle = \int_{-\infty}^{\infty} d\tau \, e^{-k(\tau - \bar{\tau})} P_G(\tau, \eta^2) = \exp\left(\frac{k^2\eta^2}{2}\right) \quad \textbf{(52)}$$

which represents the characteristic function of the Gaussian distribution.

It is clear from Eq.(32) that the reflectivity of the semi-infinite medium is independent of τ, i.e., it is independent of the stochasticity of the medium. So, formula (32) of the reflectivity has no parameters concern with the randomness of the stochastic medium.

Now, we can use the Gaussian probability density function (51) and Eq.(52) to evaluate the average values of the exponentials that appear in Eqs. (36) and (37) as

$$E_G(\nu, \bar{\tau}) = \langle \exp(-\nu\tau) \rangle = \exp\left(\frac{\nu^2\eta^2}{2} - \nu\bar{\tau}\right) \quad \textbf{(53)}$$

In terms of the statistical parameter β given by Eq.(48), Eqs.(53) can be rewritten as

$$E_G(\nu, \bar{\tau}) = \langle \exp(-\nu\tau) \rangle = \exp\left[(\nu^2\beta^2 - \nu)\bar{\tau}\right] \quad \textbf{(54)}$$

and $\bar{\tau} = \bar{\sigma} z$. It can be seen that the case of the statistical parameter $\beta = 0$ corresponds to the deterministic case. That is, there are no (or negligible) fluctuations in the medium. Therefore, increasing the value of β means that the randomness and fluctuations are increased. It is clear from Eq.(54) that if we put $\beta = 0$, these equations are reduced to the deterministic case.

Hence, the average radiant neutron energy is written as

$$\langle E(\bar{\tau}) \rangle = \frac{I_0 I_e}{I_+ - \rho^s I_-} E_G(\nu, \bar{\tau}), \quad \textbf{(55)}$$

and the average net neutron flux is

$$\left\langle F\left(\bar{\tau}\right)\right\rangle = \frac{I_0 I_f}{I_+ - \rho^s I_-} E_G\left(\nu, \bar{\tau}\right) \qquad \textbf{(56)}$$

NUMERICAL RESULTS

In this section we present some numerical calculations for the reflectivity, *R*, the average radiant neutron energy, $\left\langle E\left(\bar{\tau}\right)\right\rangle$, and the average net flux, $\left\langle F\left(\bar{\tau}\right)\right\rangle$. The externally-incident flux $\Gamma(\mu)$ is assumed to have the form

$$\Gamma\left(\mu\right) = \mu^n, \quad n = 0,1,2,\cdots \qquad \textbf{(57)}$$

We use, for calculations, three different weight functions. The used weight functions have the special forms [5], [18], [19]

$$W_1\left(\mu\right) = \mu, \qquad \textbf{(58.a)}$$

$$W_2\left(\mu\right) = \frac{\sqrt{3}}{2}\mu\left(1 + \frac{3}{2}\mu\right), \qquad \textbf{(58.b)}$$

$$W_3\left(\mu\right) = \mu I^+\left(0,\mu\right) = \mu I\left(0,-\mu\right) \qquad \textbf{(58.c)}$$

In addition, we present, graphically, the numerical results of $\left\langle E\left(\bar{\tau}\right)\right\rangle$ and $\left\langle F\left(\bar{\tau}\right)\right\rangle$ as a functions of the optical variable $\bar{\tau}$ for certain values of the statistical parameter β.

Table (1) shows the data of the reflectivity *R* for isotropic scattering, $a_3 = 0$, and transparent medium, $\rho^s = 0$, with incidence $\Gamma(\mu) = 2$ for different values of single scattering albedo ω. As we mention, in the semi-infinite medium the reflectivity is independent of the stochasticity of the random medium. So our results can be compared with that calculated by the variational method [20]. The comparison shows good agreement between the data calculated by the different weight functions and also with the published data.

Table (2) gives the reflectivity R in the case of pure-triplet scattering with $a_3 = 0.5$ for three different groups of *n* and ρ^s.

On the other side, we present, graphically, the numerical results of the average radiant neutron energy $\left\langle E\left(\bar{\tau}\right)\right\rangle$ and the average net neutron flux $\left\langle F\left(\bar{\tau}\right)\right\rangle$ as a functions of the average optical depth $\bar{\tau}$ inside the medium.

Table (1) The reflectivity *R* for isotropic scattering

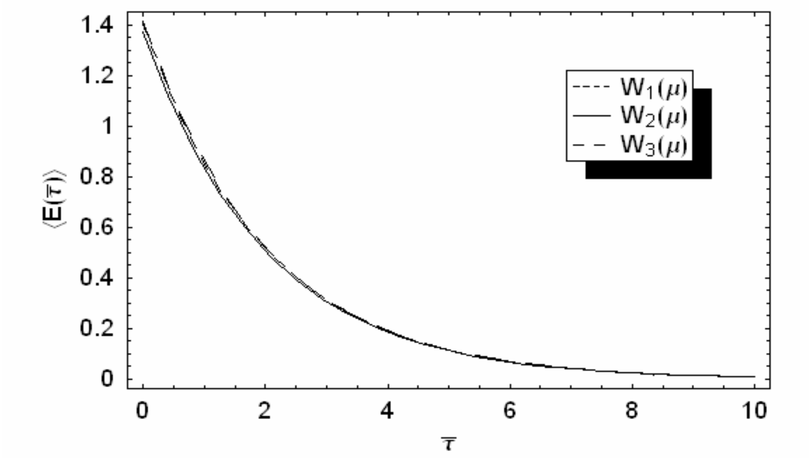
$a_3 = \rho^s = 0$, and $\Gamma(\mu) = 2$

ω	$W_1(\mu)$	$W_2(\mu)$	$W_3(\mu)$	*Ref.[20]*
0.40	0.09419	0.08180	0.10684	0.094
0.60	0.18608	0.16901	0.20180	0.182
0.80	0.35776	0.33795	0.37246	0.335
0.90	0.53137	0.51196	0.54256	0.475
0.95	0.69377	0.67627	0.70139	

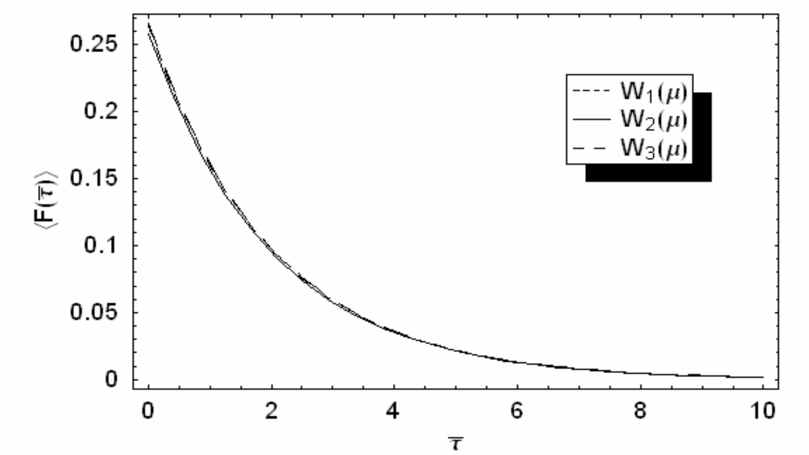
Table(2) The reflectivity R for pure-triplet scattering a_3 = 0:5

ω	$W_1(\mu)$	$W_2(\mu)$	$W_3(\mu)$
(a) n =0 and ρ^s =0.5			
0.3	0.06020	0.05399	0.05290
0.5	0.08513	0.07798	0.08243
0.7	0.14445	0.13596	0.14577
0.9	0.30857	0.29878	0.31181
(b) n =1 and ρ^s =0.5			
0.3	0.04013	0.03825	0.03642
0.5	0.05675	0.05524	0.05511
0.7	0.09630	0.09630	0.09579
0.9	0.20571	0.21163	0.20409
(c) n =1 and ρ^s =0.25			
0.3	0.03896	0.03722	0.03544
0.5	0.05444	0.05316	0.05289
0.7	0.08981	0.09021	0.08923
0.9	0.17822	0.18435	0.17646

(a) $\langle E(\bar{\tau})\rangle$

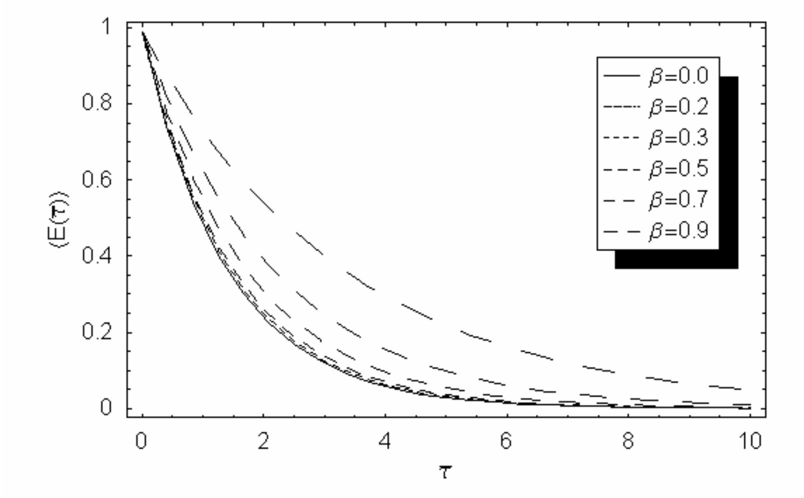


(b) $\langle F(\bar{\tau})\rangle$

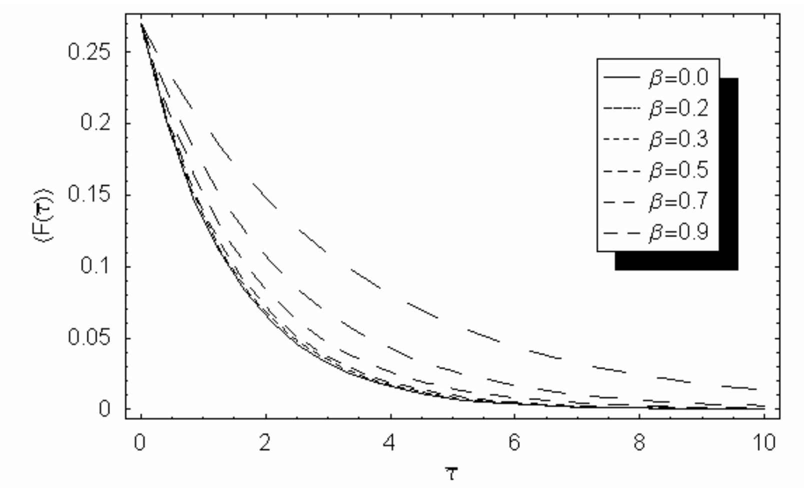


Figs.(1): (a) $\langle E(\bar{\tau})\rangle$ (b) $\langle F(\bar{\tau})\rangle$ for n=ρ^s=0, ω =0.9 & β = 0.3

(a) $\left\langle E\left(\bar{\tau}\right)\right\rangle$



(b) $\left\langle F\left(\bar{\tau}\right)\right\rangle$



Figs.(2): (a) $\langle E\left(\bar{\tau}\right)\rangle$ (b) $\langle F\left(\bar{\tau}\right)\rangle$ for n= 1, ρ^s = 0.5, and ω = 0.8

Figures Captions

1. Figures (1) give the data of (a) $\langle E\left(\bar{\tau}\right)\rangle$ and (b) $\langle F\left(\bar{\tau}\right)\rangle$ versus the average distance $\bar{\tau}$ for $n=\rho^s=0$, ω =0.9, and statistical parameter β = 0.3.

2. Figures (2) give the data of (a) $\langle E\left(\bar{\tau}\right)\rangle$ and (b) $\langle F\left(\bar{\tau}\right)\rangle$ versus $\bar{\tau}$ for n= 1, ρ^s = 0.5, and ω = 0.8 for different values of statistical parameter β, using the weight function $W_2(\mu)$

CONCLUSION

In this paper we have treated the transport of neutrons through a continuous stochastic semi-infinite medium. The medium is assumed to have specular-reflecting boundary and angular-dependent externally-incident flux. The deterministic solution is obtained at first, then the Gaussian probability density functions is used to average the solution over the medium fluctuations. We have used three different forms for the weight function, which used to force the boundary conditions to be fulfilled. One can conclude the following points from the numerical results, as given in tables or in graphical form:

1- The reflectivity in case of semi-infinite media is independent of the stochasticity of the random media, as stated before by Pomraning [3] that for the half-space problems, the boundary values at $z = 0$ are independent of the type of statistics, be Gaussian or other, adopted to describe $\sigma(z)$.

2- The results calculated by using the different three forms of the weight function are comparable with each other and give a good comparison with the published data (as shown in table (1)).

3- For higher values of β, the decay of $\langle E(\bar{\tau})\rangle$ and $\langle F(\bar{\tau})\rangle$ decreases as the optical depth variable $\bar{\tau}$ increases. This is physically acceptable since the increasing of the fluctuations of the medium (i.e. increasing of randomness), the lower decaying of $\langle E(\bar{\tau})\rangle$ and $\langle F(\bar{\tau})\rangle$.

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