

Microscopic Approach in Inelastic Heavy-Ions Scattering with Excitation of Nuclear Collective States

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In the density distribution of a deformed target-nucleus, the spherical $\lambda = 0$ and the deformed $\lambda = 2$ parts were considered. On this basis, the corresponding potential parts U_0 and $U_{int}^{(2)}$ of a double -folding microscopic nucleus-nucleus optical potential are obtained. Then, for these potentials and by using the coupled-channel technique (ECIS), the elastic and inelastic amplitudes are calculated for ^{17}O heavy ion scattering on 2^+ collective excited state of various target nuclei. Besides, the same cross-sections are calculated in the frame of an adiabatic approach of the eikonal approximation, where the inelastic amplitude is the linear function of $U_{int}^{(2)}$. Both the obtained results are compared with the experimental data, and also discuss their efficiency in predicting the deformation parameters of nuclei.

Keywords: heavy-ion microscopic optical potential, elastic and inelastic scattering theory, double folding model, high-energy approximation, ECIS code

1 Introduction

In general, it is well known the standard approaches for the calculation of the inelastic scattering of nucleons from nuclei accompanied with excitation of collective states [1]. For this purpose, one needs a transition interacting potential $U_{int}(r)$ which includes both the Coulomb and nuclear parts. The Coulomb part is obtained by using the standard method of multipole expansion, while the nuclear part can be gained by an expansion of an optical potential within the deformation addition to the nuclear radius $\delta R(\hat{r})$ such that to verify the relation, $U(R + \delta R, r) = U(R, r) + U_{int}(\mathbf{r})$. At small deformations we have, $U_{int} = (dU(r)/dR)\delta R(\hat{r})$. Further, the calculation of the inelastic scattering amplitude can be fulfilled either in the linear approximation of U_{int} , or by using the coupled channel method where all orders of U_{int} are included. In the investigation of nucleon-nucleus scattering, it is natural to consider that the radius parameter R and the deformation are the same for the target nucleus and for the scattering potential. Therefore, the deformation parameter of the potential obtained by fitting with the experimental data is concluded to be the deformation of the nucleus itself. In principle, same prescriptions are used in calculation of inelastic scattering of heavy ions. However, in so doing one should take into account some specific considerations of such task. First, in case of nucleus-nucleus scattering the deformation parameter of the potential, generally speaking, does not coincide with that of any single nucleus of these nuclei. Second, due to the charge multiplication of nuclei $Z_1 Z_2 e^2$, the Coulomb field becomes rather strong which demands to include the high orders of the transition Coulomb potential. Third, with the increase of the collision energy the usual standard methods suffer from difficulties, due to the huge number of partial waves, which will affect the preservation of the necessary accuracy. These difficulties can be overcome by using the eikonal method of the high energy approximation (HEA), which have been applied by us before on analysis of both elastic

[2,3] and inelastic [4,5] scattering of nuclei and proved itself as an effective method. The nuclear potential, as said above, was constructed in the form of a derivative of some function either phenomenological as Woods-Saxon type [4], or in the form of a microscopical folding potential of elastic scattering [5]. Thus, the received value for the deformation parameter after the comparison with experimental data keeps the character of the nucleus-nucleus potential itself. In Sec. 2 we construct the microscopical transition-density of one the target nucleus, and then the corresponding transition potential, and thus the nucleus deformation is used as a fitting parameter when compared with experimental data. Further, in Sec. 3 on the basis of this transition potential we calculate the nucleus-nucleus inelastic cross section using the code ECIS for numerical solution of the coupled channel equations as well as the adiabatic approach in the HEA model which deals with an analytic expression for inelastic scattering amplitude. In Sec. 4 a discussion is carried out of results and the possibility to use the heavy ion beams for obtaining information on the structure of colliding nuclei.

2 Folding Model of Transition Potential

When constructing the transition microscopic potential we study the excitation of the target nucleus "2". It is considered that the density distribution function $\rho_2(\mathbf{r}_2)$ includes dependence on the collective coordinates of the nucleus $\alpha_{\lambda\mu}$, which characterize coherent deformation of the radius \mathbf{r}_2 . In the case of quadrupole deformation ($\lambda = 2$), the deviation from its spherical form $\rho_2(r_2)$ can be described by changing the spherical coordinate r_2 by the coordinate \mathbf{r}_2 on the surface of an ellipsoid [1],

$$r_2 \Rightarrow \mathbf{r}_2 = r_2 + \delta\mathbf{r}_2, \quad (1)$$

$$\delta\mathbf{r}_2 = -r_2 \sum_{\mu=0,\pm 2} \alpha_{2\lambda} Y_{2\lambda}(\hat{r}_2), \quad \alpha_{2\lambda} = \beta D_{\mu 0}^{(2)*}(\Theta_i), \quad (2)$$

where \hat{r}_2 is the angular coordinates of \mathbf{r}_2 in c.m system of colliding nuclei. Here, we consider the rotation model of the nucleus and use as collective coordinates the Euler rotation angles Θ_i , while β is the corresponding deformation parameters. At $\beta \ll 1$ we keep only the linear terms in the density expansion in $\delta\mathbf{r}_2$, so that

$$\rho_2(\mathbf{r}_2) \Rightarrow \rho_2(r_2) + o_2(\mathbf{r}_2), \quad (3)$$

$$o_2(\mathbf{r}_2) = o_2^{(2)}(r_2) \sum_{\mu=0,\pm 2} \alpha_{2\mu} Y_{2\mu}(\hat{r}_2), \quad o_2^{(2)}(r_2) = -r_2 \frac{d\rho_2(r_2)}{dr_2}, \quad (4)$$

where $o_2^{(2)}(r_2)$ defines the transition density form ¹. Inserting the density given in (3) in the formulae for direct and exchange parts of the folding potential (see e.g. [3],[5]), the first term $\rho_2(r_2)$ leads to an expression for the central part of the potential, while the second one $o_2(\mathbf{r}_2)$ expresses the quadrupole transition potential as follows, ²

$$V_{int}(\mathbf{r}) = V_{int}^D(\mathbf{r}) + V_{int}^{EX}(\mathbf{r}), \quad (5)$$

¹ From a series of papers one applies the model with a deformation term $\delta\mathbf{r}_2 = -R_2 \sum \alpha_{2\lambda} Y_{2\lambda}(\hat{r}_2)$, which leads to $o_2^{(2)}(r_2) = -R_2 d\rho_2(r_2)/dr_2$. Here R_2 is considered as the radius of the nucleus, although there is no accurate definition for it. Due to this choice $o_2^{(2)}(r_2)$ usually does not fit the deformation parameter, but the "deformation length" $\delta_2 = \beta R_2$ without factorizing separate multipliers.

² In the folding potential one usually introduces the factor $F(\rho_1 + \rho_2)$ to correct dependence of nucleon-nucleon forces on the density of nuclei in their overlap region. We also consider this factor in the central part of the potential, but do not include it in the transition potential (5) because of the not so clear of its physical interpretation in the case of inelastic scattering.

where,

$$V_{int}^D(\mathbf{r}) = Cg(E) \sum_{\mu=0,\pm 2} \alpha_{2\mu} \int \rho_1(r_1) o_2^{(2)}(r_2) v_{00}^D(s) Y_{2\mu}(\hat{r}_2) d^3r_1 d^3r_2, \quad (6)$$

$$V_{int}^{EX}(\mathbf{r}) = Cg(E) 4\pi \sum_{\mu=0,\pm 2} \alpha_{2\mu} \int_0^\infty G^{(2)}(\mathbf{r}, \mathbf{s}) v_{00}^{EX}(s) j_0(K(r)s/M) s^2 ds, \quad (7)$$

$$G^{(2)}(\mathbf{r}, \mathbf{s}) = \int \rho_1(|\mathbf{u} - \mathbf{r}|) \hat{j}_1(k_{F_1}(|\mathbf{u} - \mathbf{r}|) \cdot s) o_2^{(2)}(u) Y_{2\mu}(\hat{u}) \hat{j}_1(k_{F_2}(u) \cdot s) d^3u. \quad (8)$$

Here $j_n(x)$ is the spherical Bessel function, the function $\hat{j}_1(x) = (3/x^3)(\sin x - x \cos x)$, a k_F is the Fermi momentum of a nucleon in the nucleus, and $K(r) = [(2mM/\hbar^2)(E_{cm} - V(r) - V_C(r))]$ is the local momentum of relative motion which depends on the central part of the elastic scattering potential calculated separately. In momentum representation the integrals (6), (8) transform to integrals of one dimensional form (see e.g. [6]), and the final expression for the transition density can be given in the form

$$V_{int}(\mathbf{r}) = V_{int}^{(2)}(r) \sum_{\mu=0,\pm 2} \alpha_{2\mu} Y_{2\mu}(\hat{r}), \quad (9)$$

$$V_{int}^{(2)}(r) = V_{int}^{D(2)}(r) + V_{int}^{EX(2)}(r), \quad (10)$$

where

$$V_{int}^{D(2)}(r) = Cg(E) \frac{1}{2\pi^2} \int_0^\infty \rho_1(q) o_2^{(2)}(q) v_{00}^D(q) j_2(qr) q^2 dq, \quad (11)$$

$$V_{int}^{EX(2)}(r) = 4\pi Cg(E) \int_0^\infty G^{(2)}(r, s) v_{00}^{EX}(s) j_0(K(r)s/M) s^2 ds, \quad (12)$$

$$G^{(2)}(r, s) = \int_0^\infty h_1(q, s) h_2^{(2)}(q, s) j_2(qr) q^2 dq, \quad (13)$$

$$h_1(q, s) = 4\pi \int_0^\infty \rho_1(r) \hat{j}_1(k_{F,1}(r) \cdot s) j_0(qr) r^2 dr, \quad (14)$$

$$h_2^{(2)}(q, s) = 4\pi \int_0^\infty o_2^{(2)}(x) \hat{j}_1(k_{F,2}(x) \cdot s) j_2(qx) x^2 dx, \quad (15)$$

The Fourier transform functions without/with upper index "2" are defined as follows,

$$\rho(q) = 4\pi \int_0^\infty f(r) j_0(qr) r^2 dr, \quad f_2^{(2)}(q) = 4\pi \int_0^\infty f_2^{(2)}(r) j_2(qr) r^2 dr, \quad (16)$$

3 Inelastic Scattering Cross Sections Calculations

On the basis of the above formulae we calculate microscopical transition potentials and then use them to get the cross sections of inelastic scattering for different nuclei. It is necessary that the central part of the optical potential to be known which will explain the experimental data of elastic scattering for the same colliding nuclei at the same colliding energy in the inelastic scattering channel. Calculations were performed in both cases, first by using ECIS numerical code for coupled channels [7] as well as in the adiabatic approach in HEA [4,5]. In HEA approach it is supposed an adiabatic process when the velocity of an internal nuclear motion is much smaller than the relative motion velocity of the nucleus as a whole. Then the

process can be considered as if a scattering from "frozen" nucleus with fixed collective motion coordinates $\{\alpha_{\lambda\mu}\}$. The use of HEA in this case means the calculation of eikonal amplitude f_{el} of elastic scattering, where the potential has central and quadrupole parts and depends on $\{\alpha_{\lambda\mu}\}$. The inelastic scattering amplitude is constructed in the form of a transition matrix element from initial $|00\rangle$ to final (excited) state of the nucleus $\langle IM|$ where the operator is the elastic scattering amplitude,

$$f_{in}(q) = \langle IM|f_{el}(q, \{\alpha_{\lambda\mu}\})|00\rangle \quad (17)$$

Usually in this approach there is inherent approximation when the part of the eikonal phase, $\chi_{int} = -(1/\hbar v) \int V_{int} dz$, defined through the transition potential, is considered of small value and then the corresponding part of the eikonal $\exp(i\chi_{int})$ can be expanded in a series and limit ourselves to the linear terms of V_{int} . Then using (9) the amplitude (17) can be factorized to reveal a structure factor $F_{\lambda\mu} = \langle IM|\alpha_{\lambda\mu}|00\rangle$. In this case, for a given λ , the angular distribution does not depend on the collective nature of the excited state, because the structure factor affects only the absolute value of the cross section. As is known from the nucleus rotational model, the transition structure factor $0^+ \rightarrow 2^+$ is equal to $F_{2,\mu}^{rot} = \beta(1/\sqrt{5})\delta_{M,\mu}$ which reflects that the absolute value of the cross section depends only on the deformation parameter β . All the related details of formulae can be find in [4,5]. The calculation procedure has two steps. First, a microscopic calculation for both of the real V and the imaginary W parts of the central optical potential is carried out to compute the elastic scattering cross sections. This latter is made with the help of code ECIS [7] and also using the HEA model the differential cross sections, and then they are compared with the experimental data. During the fitting process a variation will be done for evaluating the strength parameters N_r and N_{im} which define contributions of the real V and the imaginary W parts into microscopic optical potentials. As a result we denote optical potentials, in the elastic channel as follows,

$$U_{opt}(r) = N_r V(r) + iN_{im} W(r). \quad (18)$$

The elastic scattering of ^{17}O nucleus from ^{60}Ni , ^{90}Zr , ^{120}Sn and ^{208}Pb nuclei at energy $E_{lab}=1435$ MeV was studied previously by us [5] using corresponding potentials in HEA case and results were compared with experimental data given in Ref [8]. Similar calculations by using ECIS code are given in [9]. The real parts of these micro-potentials are calculated using the double folding model which includes an exchange term with the effective nucleon-nucleon Paris potential in the form of the CDM3Y6 dependence on density [6](see also [5]). As to the imaginary part W^H , it was calculated first in the frame of microscopic approach, given in details in Ref [3], on the basis of the Glauber and Sitenko multiple scattering theory [10,11]. In addition, in [3] it was used for the imaginary part, as another version, the form of the double folding real part V^{DF} . It was clear from all these different types of calculations, that by fitting the strength parameters N_r and N_{im} one can get the respective cross sections in a good agreement with the experimental data. However, in our present consideration we prefer to apply to the double folding transition potential V_{int} as the sample for the imaginary potential when fitting the normalizing strength parameters N_r and N_{im} . So in the following we use the microscopic transition potential in the form,

$$U_{int}(r) = N_r V_{int}(r) + iN_{im} V_{int}(r), \quad (19)$$

where V_{int} potential is calculated by using Eqs.(9)-(15). In Fig. 1 it is shown the radial dependence of the transition potentials $N_r V_{int}^{(2)}$, and also the central potentials of elastic

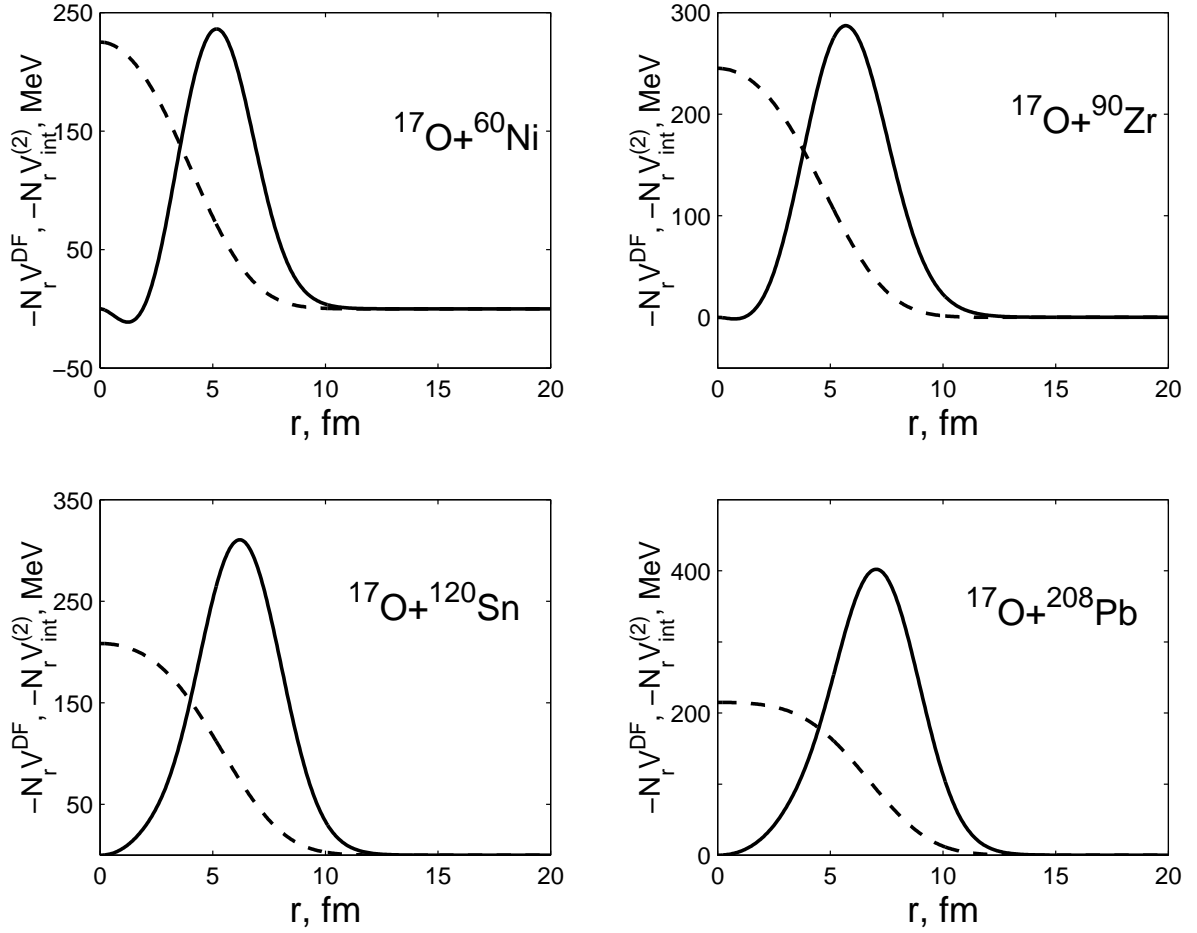


Fig. 1. Real part of potentials $N_r V^{DF}(r)$ of elastic (dashed lines) and $N_r V_{int}^{(2)}(r)$ of inelastic (solid lines) nucleus-nucleus interaction, for the scattering of heavy ions ^{17}O from different target nuclei at $E_l = 1435$ MEV(see text).

scattering $N_r V^{DF}$. In concern of the transition Coulomb potentials, they were calculated as usual for the interaction of a charge $Z_1 e$ with the field of uniformly distributed charge $Z_2 e$ in ellipsoid volume with radius $R_c (1 + \sum \alpha_{2\mu} Y_{2\mu}(\hat{r}))$, where $R_c = 1.2 (A_1^{1/3} + A_2^{1/3})$ fm. The corresponding relations for central and quadrupole transition Coulomb potentials and also for the eikonal phases are given in [4].

4 Comparison with Experimental Data, Discussion, and Conclusions

In Fig. 2 it is shown the theoretical calculations for the differential cross sections of inelastic scattering using the code ECIS (solid lines) and HEA (dotted lines). The elastic scattering potential therewith is calculated, as said above, by fitting the strength parameters N_r and N_{im} to the experimental data without coupling between the elastic and inelastic channels. Next step is calculations of inelastic scattering by fitting only the deformation parameter β for the target nucleus. It is to be noted that when using the ECIS code for inelastic scattering the coupling of elastic and inelastic channels was involved through the transition potential, and

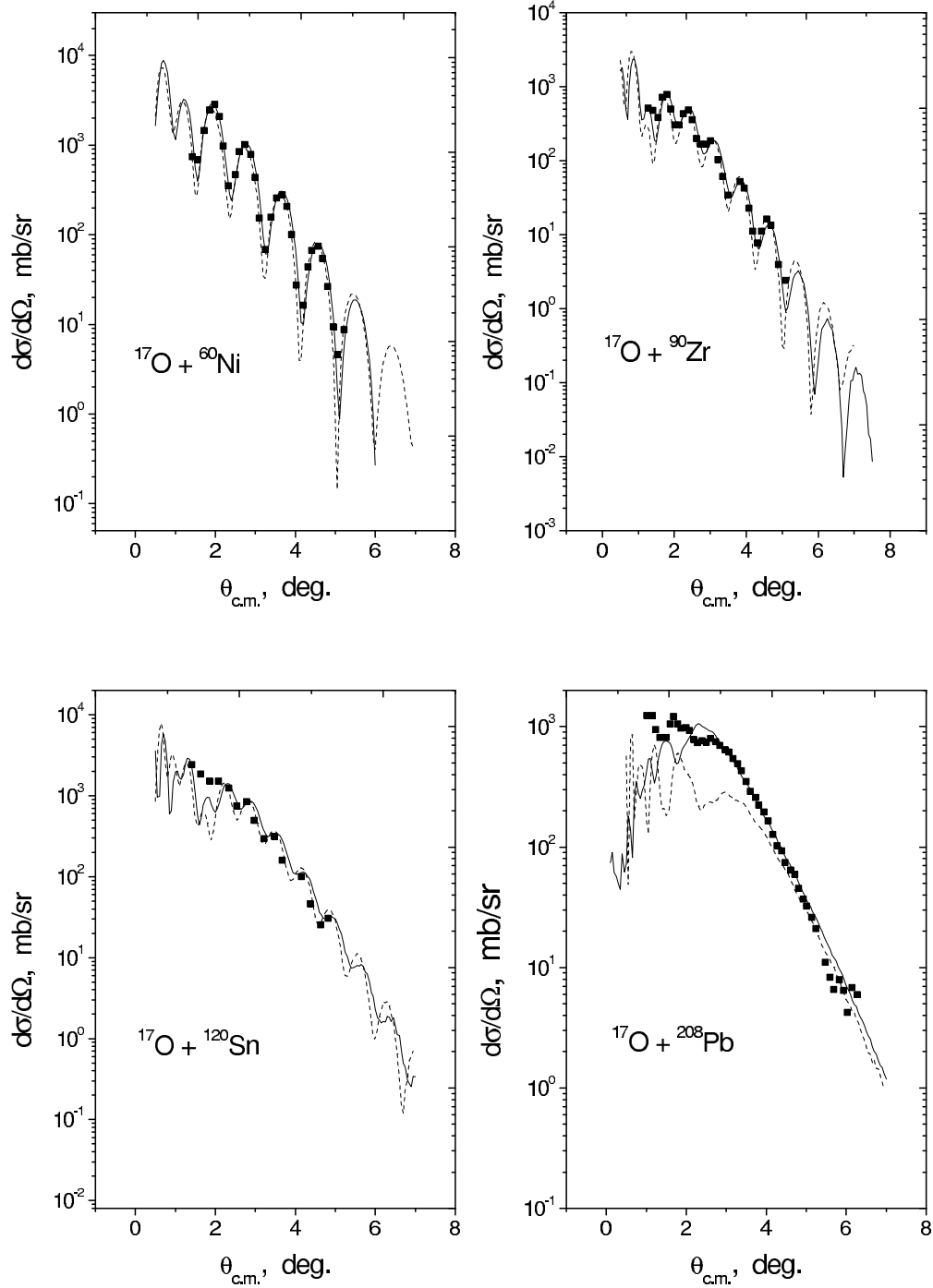


Fig. 2. Inelastic differential cross section, calculated by using coupled channels method (solid lines) and HEA method(dashed lines) for interaction of nucleus ^{17}O with target nuclei at 1435 MeV with excitation of transitions $0^+ \rightarrow 2^+$. Experimental data from [8].

Table. Renormalizing Parameters N_r, N_{im} , Coulomb Deformation $\beta^{(c)}$ and $\beta^{(n)}$, and Nuclear Transition Potentials Parameters

Parameters		$^{17}\text{O}+^{60}\text{Ni}$	$^{17}\text{O}+^{90}\text{Zr}$	$^{17}\text{O}+^{120}\text{Sn}$	$^{17}\text{O}+^{208}\text{Pb}$
Renormalizing coefficients	N_r	0.6	0.6	0.5	0.5
	N_{im}	0.6	0.5	0.5	0.8
Coulomb deformation	$\beta^{(c)}$	0.2067	0.091	0.1075	0.0544
nuclear deformation (ECIS)	$\beta^{(n)}$	0.2541	0.071	0.1063	0.078*
nuclear deformation (HEA)	$\beta^{(n)}$	0.4	0.14	0.25	0.12

* For the imaginary part value of the transition density $\beta_{im}^{(n)} = 0.0222$

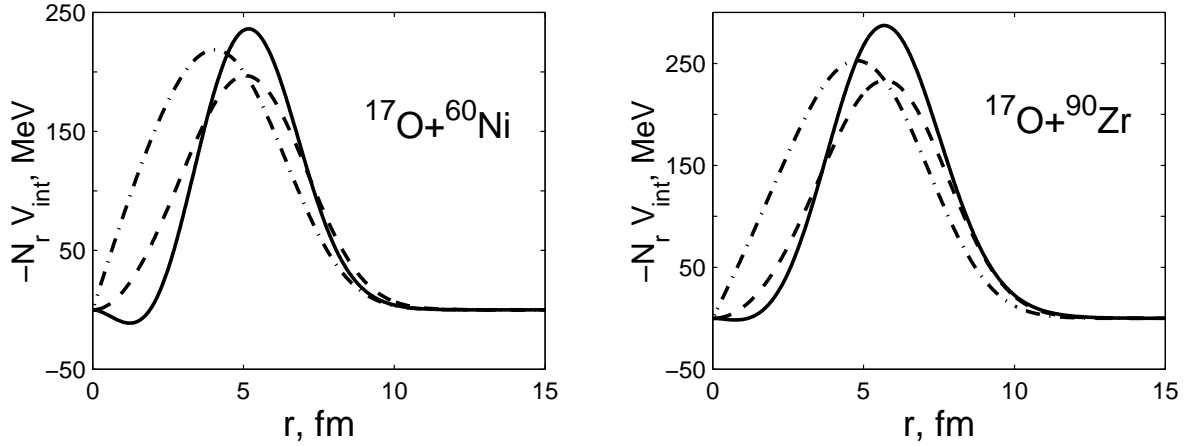


Fig. 3. Transition potentials (radial part) for inelastic scattering of heavy ions ^{17}O from nuclei ^{60}Ni and ^{90}Zr at 1435 MeV. Microscopic calculations $N_r V_{int}^{(2)}(r)$ – solid lines. Semi-microscopic model : $V_{int}^{DF-I}(r)$ (dashed) and $V_{int}^{DF-II}(r)$ (dash dotted) (see text).

its contribution was taken in all orders of perturbation theory. This contribution is defined by the deformation parameter in the density distribution function $\beta^{(n)}$ of the target nucleus and by the deformation $\beta^{(c)}$ of the unified charge density distribution in a sphere with radius as the sum of radii of colliding nuclei. In general, this parameter must not coincide with the deformation parameter of the charge distribution of the target nucleus. Nevertheless, we get the value of $\beta^{(c)}$ from the data of electric transition probabilities to 2^+ level for different target nuclei as given in Ref.[8]. These values are done in the relevant Table. The second parameter $\beta^{(n)}$ was fitted to the experimental data of inelastic scattering. As to the HEA calculations of cross sections, these parameters are involved as the linear terms of the transition potentials in the inelastic scattering amplitude. We notice from Fig. 2 that in both cases one gets reasonable good agreement with experimental data, except at the small scattering angles in the HEA calculations. The received results for the deformation parameters $\beta^{(n)}$ in the two methods differ in 1.5 - 2 times from each others. In all cases except the scattering on ^{208}Pb nucleus in ECIS calculations, the $\beta^{(n)}$ parameters for real and imaginary parts of the transition potentials are equal. As an exceptional case we were obliged to perform fit with

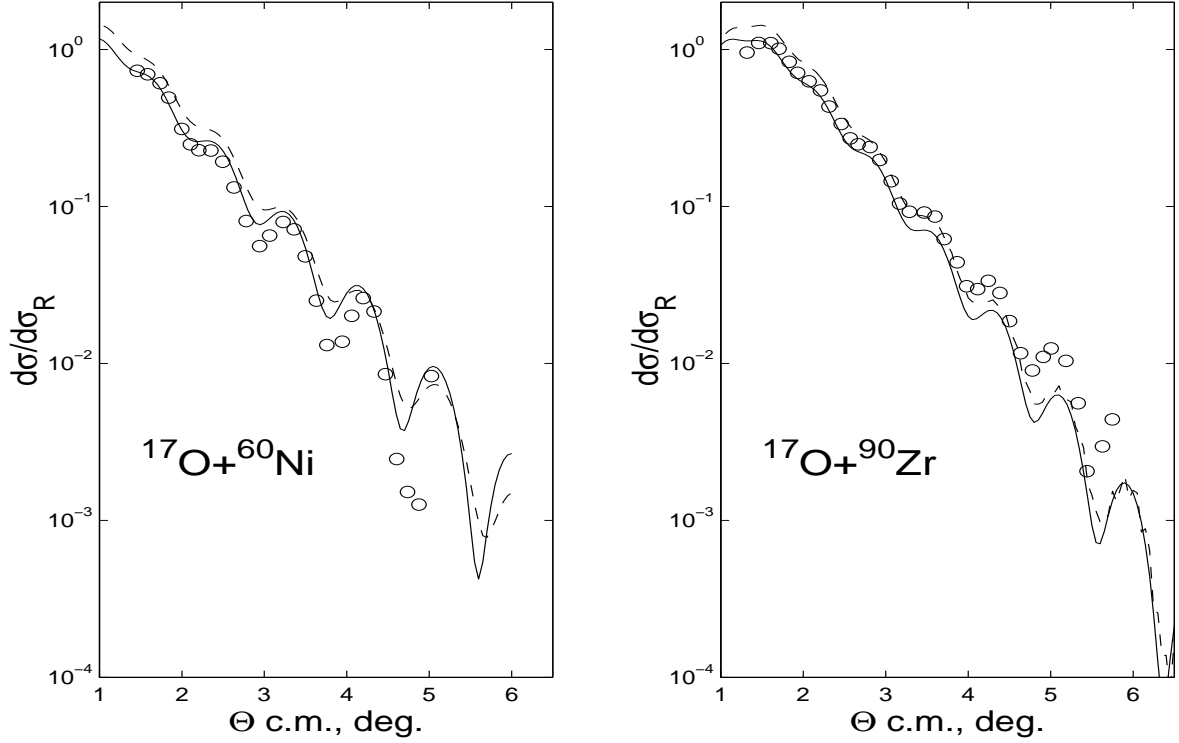


Fig. 4. The ratio of elastic differential cross sections to the Rutherford ones for scattering of heavy ions ^{17}O on nuclei ^{60}Ni and ^{90}Zr at 1435 MeV without coupled channel effects $0^+ \rightarrow 2^+$ (solid lines) and with this effect (dashed lines). Experimental data from [8].

different deformation parameters for both of the real and imaginary parts of the transition potentials and we received the following values $\beta_r^{(n)}=0.078$ and $\beta_{im}^{(n)}=0.0222$. However, it is conceivable that this result can be changed in case if one changes the transition density $o_2^{(2)}$ from the form $r_2(d\rho_2(r_2)/dr_2)$ to the form $R_2(d\rho_2(r_2)/dr_2)$, where R_2 is the radius of the target nucleus. In fact, this leads to enlarge the contribution of the Coulomb transition potential in the peripheral region, and therefore one does not enhance the nuclear part of the amplitude at the sacrifice of the imaginary part of the transition nuclear potential. This is shown in Fig. 3 where it is given three transition potentials, two of them (dashed and dotted dashed) correspond to the transition potentials constructed from derivatives of the microscopic potentials of elastic scattering in the forms $V_{int}^{DF-I} = N_r r (dV^{DF}(r)/dr)$ and $V_{int}^{DF-II} = N_r R (dV^{DF}(r)/dr)$, where $R = r_m$ is the radius at the maximum point of the function V_{int}^{DF-I} . In the peripheral area, the second transition potential occurs lower in comparison with the first one, giving the advantage to the Coulomb transition potential content, which illustrates the above mentioned comments. The third solid curve is our microscopic potential $N_r V_{int}^{(2)}$ and we see the difference in its behavior in comparison with the other two potentials constructed on the basis of semi-microscopic approach where the transition potentials are constructed as the derivatives of elastic scattering potentials.

In conclusion, we denote that it was better to define simultaneously all the three parameters N_r , N_{im} , and $\beta^{(n)}$ in the process of fitting for getting the cross sections of elastic and inelastic scattering, in place of dividing the problem into two steps, the first to get the strength parameters N_r and N_{im} in the elastic channel and then the $\beta^{(n)}$ parameter in the inelastic channel. As an example, in Fig. 4 the solid lines show the differential cross sections of elastic

scattering when the micro-potentials the N_r and N_{im} parameters are from the Table while the dashed line shows the cross section when we use the same potentials and simultaneously takes into account the effect on elastic scattering of the inelastic channel in the ECIS code. The effect of inelastic channel does not give, in principle, an observable change of the angular distribution in elastic cross sections. In view of the above study it can be seen, in general, the ways of the further study of mechanisms of nucleus-nucleus inelastic scattering to receive more accurate data about the deformation parameters and also on the transition structure matrix elements.

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