Time-Harmonic Generalized Electromagnetic fields and inhomogeneous media

P. S. Bisht and O. P. S. Negi

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Department of Physics
Kumaun University
S. S. J. Campus
Almora- 263601(Uttartakhand), India

Email: ps_bisht123@rediffmail.com
ops_negi@yahoo.co.in

Abstract

Quaternion analysis of time dependent Maxwell's equations in presence of electric and magnetic charges has been developed in unique, simple and consistent manner. It has been shown that this theory is extended consistently to time-harmonic Maxwell's equation for dyons. Reformulation of the generalized electromagnetic fields of dyons in inhomogeneous media and corresponding quaternionic equations are derived in compact, simple and unique manner. We have also discussed the monochromatic fields of generalized electromagnetic fields of dyons in slowly changing media in a consistent manner.

1 Introduction

Physicists were fascinated about magnetic monopoles since its ingenious idea was put forward by Dirac [1] and also by Saha [2]. So many attempts [3, 4] were made for the experimental verification of conclusive existence of magnetic monopoles and after the failure of attempts, the literature [5, 6, 7] turned partially negative casting doubts on the existence of such particles. The work of the Schwinger [8]was the first exception to the argument against the existence of monopoles. At the same time so many paradoxes were related to the theory of pure Abelian monopoles, as Dirac's veto [1, 2], wrong spin-statistics connection [9] and many others [10, 11]. Several problems were soon resolved by the invention of dyons [12, 13, 14, 15] particles carrying simultaneous existence of electric and magnetic charges. Fresh interest in this subject was enhanced by the idea given by t' Hooft [16] and Polyakov [2] showing that monopoles are the intrinsic parts of grand unified theories. The Dirac monopoles is an elementary particle but the t' Hooft - Polyakov monopoles [16, 2] is complicated extended object having a definite mass and finite size inside of which massive fields play a role in providing a smooth structure and outside it they vanish rapidly leaving the field configuration identical to abelian Dirac monopole. Julia and Zee [18] have extended the idea of t' Hooft [16] and Polyakov [2] to construct the classical solutions for non - Abelian dyons. Kravchenko and co-authors [19, 20], discussed the Maxwell's equations in homogeneous media and accordingly developed [21] the quaternionic reformulation of the time-dependent Maxwell's equations along with the classical solution of a moving source i.e. electron. Kravchenko et al have also demonstrated [22] the electromagnetic fields in chiral media and their quaternionic form in a simple and consistent manner. Recently, the work of Kravchenko [19, 20] is extended and GDM equations in homogeneous (isotropic) medium are discussed [23] while their quaternionic forms in a unique and consistent way are developed [24]. Keeping in view all these facts in mind, in this paper we have undertaken the study of quaternion analyticity of time harmonic Maxwell's equations of generalized electromagnetic fields in presence of electric and magnetic sources (i. e. dyons). We have also derived the generalized theory of Maxwell's - Dirac equation in presence of electric and magnetic charges in homogeneous media. It has been shown that the quantum equations and equation of motion represent the dynamics of electric charge similar to the theory described by Kravchenko [19, 20] in the absence of magnetic monopoles or vice-versa . Finally, we have reformulated the generalized electromagnetic fields of dyons in inhomogeneous media along with its quaternionic reformulations and corresponding quantum equations are derived consistently. At last, we have discussed the monochromatic fields of generalized electromagnetic fields of dyons in slowly changing media in consistent way .

2 Dyonic field equation in homogenous (isotropic) medium

Assuming the existence of magnetic monopoles and taking the case of homogenous (isotropic) medium we have already derived the generalized Maxwell's-Dirac field equations in the following form [25];

$$\begin{aligned}\vec{\nabla}.\vec{E} &= \frac{\rho_e}{\epsilon}\\ \vec{\nabla}.\vec{B} &= \mu\rho_m\end{aligned}$$

$$\begin{aligned}\vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t}-\frac{\vec{j_m}}{\epsilon} \\ \vec{\nabla}\times\vec{B} &= \frac{1}{v^2}\frac{\partial\vec{E}}{\partial t}+\mu\vec{j_e} \qquad (1)\end{aligned}$$

where ρ_e and ρ_m are respectively the electric and magnetic charge densities while $\vec{j_e}$ and $\vec{j_m}$ are the corresponding current densities, $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and ϵ and μ are defined respectively as relative permitivity and permeability in electric and magnetic fields. Differential equations (1) are the generalised field equations of dyons in homogenous (isotropic) medium and the electric and magnetic fields are corresponding called generalized electromagnetic fields of dyons. These electric and magnetic fields of dyons are expresed in following differential form in homogenous (isotropic) medium in terms of two four - potentials as,

$$\vec{E} = -\vec{\nabla}\phi_e-\frac{\partial\vec{C}}{\partial t}-\vec{\nabla}\times\vec{D} \qquad (2)$$

$$\vec{B} = -\vec{\nabla}\phi_m-\frac{1}{v^2}\frac{\partial\vec{D}}{\partial t}+\vec{\nabla}\times\vec{C} \qquad (3)$$

where $\{C^\mu\} = \{\phi_e, \overrightarrow{vC}\}$ and $\{D^\mu\} = \{v\phi_m, \vec{D}\}$ are the two four-potentials associated with electric and magnetic charges. Let us define the complex vector field $\vec{\psi}$ in the following form [25]

$$\vec{\psi} = \vec{E}-iv\vec{B}. \qquad (4)$$

Equations (2,3) and (4) leads the following relation between generalized field and the components of generalized four-potential as,

$$\vec{\psi} = -\frac{\partial\vec{V}}{\partial t}-\vec{\nabla}\phi-iv(\vec{\nabla}\times\vec{V}) \qquad (5)$$

where $\{V_\mu\}$ is the generalized four - potential of dyons in homogenous (isotropic) medium and defined as

$$V_\mu = \{\phi, \vec{V}\} \qquad (6)$$

i.e.

$$\phi = \phi_e-iv\phi_m \qquad (7)$$

and

$$\vec{V} = \vec{C}-i\frac{\vec{D}}{v}. \qquad (8)$$

Maxwell's field equation (1) in isotropic medium may then be written in terms of generalized field $\overrightarrow{\psi}$ as

$$\begin{aligned} \overrightarrow{\nabla}\cdot\overrightarrow{\psi} &= \frac{\rho}{\epsilon} && (9) \\ \overrightarrow{\nabla}\times\overrightarrow{\psi} &= -iv(\mu\overrightarrow{j}+\frac{1}{v^2}\frac{\partial\overrightarrow{\psi}}{\partial t}) && (10) \end{aligned}$$

where ρ and $\overrightarrow{j}$ the generalized charge and current source densities of dyons in homogenous medium given by [25]

$$\begin{aligned} \rho &= \rho_e - i\frac{\rho_m}{v} && (11) \\ \overrightarrow{j} &= \overrightarrow{j_e} - iv\overrightarrow{j_m}. && (12) \end{aligned}$$

In terms of complex potential the equation is written as

$$\begin{aligned} \Box\phi &= v\mu\rho && (13) \\ \Box\overrightarrow{V} &= \mu\overrightarrow{j} && (14) \end{aligned}$$

We write the following tensorial form of generalized Maxwell's -Dirac equations of dyons in homogenous (isotropic) medium as

$$\begin{aligned} F_{\mu\nu,\nu} &= j_\mu^e && (15) \\ F_{\mu\nu,\nu}^d &= j_\mu^m. && (16) \end{aligned}$$

Defining generalized field tensor of dyon as

$$G_{\mu\nu} = F_{\mu\nu} - ivF_{\mu\nu}^d. \qquad (17)$$

One can directly obtain the following generalized field equation of dyon in homogenous (isotropic) medium i.e.

$$\begin{aligned} G_{\mu\nu,\nu} &= j_\mu && (18) \\ G_{\mu\nu,\nu}^d &= 0. && (19) \end{aligned}$$

3 Quaternion Analyticity of time harmonic dyonic field equation

Using the Fourier transform any electromagnetic field can be represented as an infinite superposition of time-harmonic (monochromatic) fields. These fields are normally the main object of study in radio electronics, wave propagation theory and many other branches of physics and engineering. A time harmonic electromagnetic field has the following form [19]

$$\overrightarrow{E}(x,t) = Re(\overrightarrow{E}(x)e^{-i\omega t}) \tag{20}$$

and

$$\overrightarrow{B}(x,t) = Re(\overrightarrow{B}(x)e^{-i\omega t}) \tag{21}$$

where the electric field $\overrightarrow{E}$ and magnetic field $\overrightarrow{B}$ depend on the spatial variables $x = (x_1, x_2, x_3)$ and all dependance on time is contained in the factor $e^{-i\omega t}$, $\overrightarrow{E}$ and $\overrightarrow{B}$ are complex vectors called the complex amplitudes of electromagnetic field and ω is the frequency of oscillations.

Substituting the values of $\overrightarrow{E}$ and $\overrightarrow{B}$ into the generalized dyonic equation (1) in isotropic medium, we obtain

$$\begin{aligned}
\overrightarrow{\nabla}.\overrightarrow{E} &= \frac{\rho_e}{\epsilon} \\
\overrightarrow{\nabla}.\overrightarrow{B} &= \mu\rho_m \\
\overrightarrow{\nabla}\times\overrightarrow{E} &= -i\omega\overrightarrow{B} - \frac{\overrightarrow{j_m}}{\epsilon} \\
\overrightarrow{\nabla}\times\overrightarrow{B} &= -\frac{i\omega}{v^2}\overrightarrow{E} + \mu\overrightarrow{j_e}.
\end{aligned} \tag{22}$$

Let us denote $\alpha = \omega\sqrt{\epsilon\mu} = \frac{\omega}{v}$, where the square root is chosen that $Im\alpha \geq 0$, The quantity α is called the wave number. Let us write the D, E and B in the quaternionic form as,

$$D = \partial_1 e_1 + \partial_2 e_2 + \partial_3 e_3 \tag{23}$$
$$E = E_1 e_1 + E_2 e_2 + E_3 e_3 \tag{24}$$
$$B = B_1 e_1 + B_2 e_2 + B_3 e_3 \tag{25}$$

where e_1, $e_2$and e_3 are the quaternions and satisfy the following multiplication rule,

$$\begin{aligned}
e_0^2 &= 1 \\
e_j e_k &= -\delta_{jk} + \epsilon_{jkl} e_l
\end{aligned} \tag{26}$$

where δ_{jk} and ϵ_{jkl} (j, k, l= 1,2,3 and $e_0 = 1$) are respectively the Kronecker delta and three-index Levi-Civita symbol. Taking the quaternionic form of equation (21) as,

$$
\begin{aligned}
D\overrightarrow{E} &= (\partial_1 e_1 + \partial_2 e_2 + \partial_3 e_3)(E_1 e_1 + E_2 e_2 + E_3 e_3) \\
&= -\tfrac{\rho_e}{\epsilon} \quad -\frac{\overrightarrow{j_m}}{\epsilon} + i\omega\overrightarrow{B}
\end{aligned} \tag{27}
$$

and

$$
D\overrightarrow{B} = -\mu\rho_m + \mu\overrightarrow{j_e} - i\frac{\omega}{v^2}\overrightarrow{E}. \tag{28}
$$

Let us introduce the following pair of purely vectorial biquaternionic functions i.e.

$$
\overrightarrow{l} = -\frac{i\omega}{v^2}\overrightarrow{E} + \alpha\overrightarrow{B} \tag{29}
$$

and

$$
\overrightarrow{m} = \frac{i\omega}{v^2}\overrightarrow{E} + \alpha\overrightarrow{B}. \tag{30}
$$

Taking the divergance of third and fourth equation (22), we get the following pair of continuity equation for electric and magnetic charge is,

$$
\overrightarrow{\nabla}.\overrightarrow{j_e} - i\omega\rho_e = 0 \tag{31}
$$

and

$$
\overrightarrow{\nabla}.\overrightarrow{j_m} - i\omega\mu\epsilon\rho_m = 0. \tag{32}
$$

Applying the quaternionic operator D to l and using equations (27,28) and (31,32), we find

$$
D\overrightarrow{l} = \frac{1}{\epsilon v^2}[\overrightarrow{\nabla}.\overrightarrow{j}*] + \alpha\mu\overrightarrow{j}* + \alpha\overrightarrow{l} \tag{33}
$$

where $\overrightarrow{j}*$ is the complex conjugate of dyonic current density in homogenous (isotropic) medium given by equation (12). Thus $\overrightarrow{l}$ satisfies the equation which is derived by equation (33) as,

$$
(D-\alpha)\overrightarrow{l} = \mu[\overrightarrow{\nabla}.\overrightarrow{j}*] + \alpha\mu\overrightarrow{j}*. \tag{34}
$$

Analogous to equation (34), $\overrightarrow{m}$ satisfies the equation (34) as,

$$(D+\alpha)\overrightarrow{m} = -\mu[\overrightarrow{\nabla}.\overrightarrow{j}] + \alpha\mu\overrightarrow{j}. \tag{35}$$

Thus, the prodecure of diagonalization can be written in the matrix form

$$\begin{pmatrix} D & -i\omega \\ i\omega & D \end{pmatrix}\begin{pmatrix} \overrightarrow{E} \\ \overrightarrow{B} \end{pmatrix} = B_\alpha \begin{pmatrix} D-\alpha & 0 \\ 0 & D+\alpha \end{pmatrix} B_\alpha^{-1} \begin{pmatrix} \overrightarrow{E} \\ \overrightarrow{B} \end{pmatrix} \tag{36}$$

where

$$B_\alpha = \begin{pmatrix} -\frac{i\omega}{v^2} & \alpha \\ \frac{i\omega}{v^2} & \alpha \end{pmatrix} \tag{37}$$

and

$$B_\alpha^{-1} = \begin{pmatrix} -\frac{v^2}{i\omega} & \frac{v^2}{i\omega} \\ \frac{1}{\alpha} & \frac{1}{\alpha} \end{pmatrix}. \tag{38}$$

Here we obtain two decoupled equations for the unknown vectors $\overrightarrow{l}$ and $\overrightarrow{m}$, which simplifies the analysis of the generalized Dirac - Maxwell's (GDM) equation of dyons in homogenous (isotropic) medium.

4 Dyons in inhomogeneous medium

In order to discuss the GDM equation in inhomogeneous medium, let us assume ε and μ are the function of coordinates [26] such as

$$\varepsilon = \varepsilon(x)$$

and

$$\mu = \mu(x). \tag{39}$$

The Maxwell's equations are thus considered together with the relation given by (39) may then be written accordingly in inhomogeneous media describing relations among the induction and the field vectors. Let us now define $\overrightarrow{D}$ and $\overrightarrow{B}$ as,

$$\overrightarrow{D} = \overrightarrow{D}(\overrightarrow{E},\overrightarrow{H})$$

$$\overrightarrow{B} = \overrightarrow{B}(\overrightarrow{E},\overrightarrow{H}) \tag{40}$$

and

$$\overrightarrow{D} = \varepsilon_0 \varepsilon_r \overrightarrow{E}$$

$$\overrightarrow{B} = \mu_0 \mu_r \overrightarrow{H}. \tag{41}$$

Using the relation (40), we may write the generalized Dirac - Maxwell (GDM) equations (1) for the inhomogenous medium for dyons as

$$\overrightarrow{\nabla}.(\varepsilon \overrightarrow{E}) = \rho_e$$

$$\overrightarrow{\nabla}.(\overrightarrow{B}) = \mu \rho_m$$

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} - \frac{\overrightarrow{j_m}}{\epsilon}$$

$$\overrightarrow{\nabla} \times \overrightarrow{B} = \tfrac{1}{v^2} \frac{\partial \overrightarrow{E}}{\partial t} + \mu \overrightarrow{j_e} \tag{42}$$

The first and second differential equations of GDM equations (42) can be written as follows [19] ,

$$\overrightarrow{\nabla}.\overrightarrow{E} + < \tfrac{\overrightarrow{\nabla}\varepsilon}{\varepsilon}, \overrightarrow{E} >= \frac{\rho_e}{\varepsilon} \tag{43}$$

and

$$\overrightarrow{\nabla}.\overrightarrow{B} + < \tfrac{\overrightarrow{\nabla}\mu}{\mu}, \overrightarrow{B} >= \mu \rho_m. \tag{44}$$

Combining equations (43) and (44) with the third and fourth equation of (42), we obtain the following form of generalized Dirac - Maxwell (GDM) equations i.e.

$$\overrightarrow{\nabla}.\overrightarrow{E} = < \tfrac{\overrightarrow{\nabla}\varepsilon}{\varepsilon}, \overrightarrow{E} > - \frac{\partial \overrightarrow{B}}{\partial t} - \frac{\overrightarrow{j_m}}{\varepsilon} - \frac{\rho_e}{\varepsilon} \tag{45}$$

and

$$\overrightarrow{\nabla}.\overrightarrow{B} = < \tfrac{\overrightarrow{\nabla}\mu}{\mu}, \overrightarrow{B} > \frac{1}{v^2} \frac{\partial \overrightarrow{E}}{\partial t} - \mu \overrightarrow{j_e} - \mu \rho_m. \tag{46}$$

Let us take the scalar products of two vectors $\overrightarrow{p}$ and $\overrightarrow{q}$ as,

•

$$< \overrightarrow{p}, \overrightarrow{q} > = -\frac{1}{2}(\overrightarrow{p} M + M \overrightarrow{p}) \overrightarrow{q}. \qquad (47)$$

Using this equation (47), we get the following pair of equations from equations (45) and (46) as

$$(D + \frac{1}{2}\frac{\overrightarrow{\nabla}\varepsilon}{\varepsilon})\overrightarrow{E} = -\frac{1}{2}M\frac{\overrightarrow{\nabla}\varepsilon}{\varepsilon}\overrightarrow{E} - \frac{\partial \overrightarrow{B}}{\partial t} - \frac{\overrightarrow{j_m}}{\varepsilon} - \frac{\rho_e}{\varepsilon} \qquad (48)$$

and

$$(D + \frac{1}{2}\frac{\overrightarrow{\nabla}\mu}{\mu})\overrightarrow{B} = -\frac{1}{2}M\frac{\overrightarrow{\nabla}\mu}{\mu}\overrightarrow{B} - \frac{1}{v^2}\frac{\partial \overrightarrow{E}}{\partial t} - \overrightarrow{\mu j_e} - \mu\rho_m. \qquad (49)$$

where we have used the following subsidiary condition

$$\frac{1}{2}\frac{\overrightarrow{\nabla}\varepsilon}{\varepsilon} = \frac{\overrightarrow{\nabla}\sqrt{\varepsilon}}{\sqrt{\varepsilon}} \qquad (50)$$

As such, on using the condition given by Kravehenko [19], we may write equations (48) and (49) as follows,

$$\frac{1}{\sqrt{\varepsilon}}D(\sqrt{\varepsilon}.\overrightarrow{E}) + \overrightarrow{E}.\overrightarrow{\varepsilon} = -\frac{\partial \overrightarrow{B}}{\partial t} - \frac{\overrightarrow{j_m}}{\varepsilon} - \frac{\rho_e}{\varepsilon} \qquad (51)$$

$$\frac{1}{\sqrt{\mu}}D(\sqrt{\mu}.\overrightarrow{B}) + \overrightarrow{B}.\overrightarrow{\mu} = -\frac{1}{v^2}\frac{\partial \overrightarrow{E}}{\partial t} - \overrightarrow{\mu j_e} - \mu\rho_m \qquad (52)$$

where

$$\overrightarrow{\varepsilon} = \frac{\overrightarrow{\nabla}\sqrt{\varepsilon}}{\sqrt{\varepsilon}}$$

and

$$\overrightarrow{\mu} = \frac{\overrightarrow{\nabla}\sqrt{\mu}}{\sqrt{\mu}}.$$

Rearranging equations (51) and (52), we get the following new set of equations i.e.

$$(D + M^{\overrightarrow{\varepsilon}})\overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} - \frac{\overrightarrow{j_m}}{\varepsilon} - \frac{\rho_e}{\varepsilon} \quad (53)$$

$$(D + M^{\overrightarrow{\mu}})\overrightarrow{B} = -\frac{1}{v^2}\frac{\partial \overrightarrow{E}}{\partial t} - \overrightarrow{\mu j_e} - \mu\rho_m. \quad (54)$$

In order to write the quaternionic equation for electromagnetic fields in inhomogeneous medium, let us start with the following representations for the quaternionic differential operator [23],

$$\boxdot = (\frac{i}{v}\partial_t + D) \quad (55)$$

and its conjugate as

$$\overline{\boxdot} = (\frac{i}{v}\partial_t + D) \quad (56)$$

where $v = \sqrt{\frac{1}{\varepsilon\mu}}$ is the speed of the electromagnetic wave in the medium. We define the complex vector field $\overrightarrow{\psi}$ associated with the generalized electromagnetic fields of dyons as ,

$$\overrightarrow{\psi} = \overrightarrow{E} - iv\overrightarrow{B}. \quad (57)$$

The quantum equation associated with the generalized four-current is described in the following form i. e.

$$j = -i\rho v + j_1 e_1 + j_2 e_2 + j_3 e_3 \quad (58)$$

Operating equation (56) to equation (57) and using equations (53) and (26), we get

$$\overline{\boxdot}\psi = (M^{\overrightarrow{\varepsilon}}\overrightarrow{E} + ivM^{\overrightarrow{\mu}}\overrightarrow{B}) + iv\mu(-iv\overline{\rho} + \overrightarrow{j}) \quad (59)$$

where $\overrightarrow{j}$ is the generalized current density of dyon and is given by equation (12) and $\overline{\rho}$is the conjugate of the generalized charge density of dyon and defined by equation (11). From equation (57), we get the following relations,

$$\overrightarrow{E} = \frac{1}{2}(\psi + \psi*) \quad (60)$$

and

$$\overrightarrow{B} \quad = \quad \frac{1}{2iv}(\psi * - \psi) \tag{61}$$

where $\psi*$ is the complex conjugate of ψ. Taking the first part of the right hand side of equation (59), we get

$$M^{\overrightarrow{\varepsilon}}\overrightarrow{E} + ivM^{\overrightarrow{\mu}}\overrightarrow{B} \quad = \quad \frac{1}{2}(M^{(\overrightarrow{\varepsilon}-\overrightarrow{\mu})}\psi + M^{(\overrightarrow{\varepsilon}+\overrightarrow{\mu})}\psi*). \tag{62}$$

We may now introduce the notation

$$\overrightarrow{\varepsilon} + \overrightarrow{\mu} \quad = \quad -\frac{\overrightarrow{\nabla} v}{v} \tag{63}$$

and

$$\overrightarrow{\varepsilon} - \overrightarrow{\mu} \quad = \quad -\frac{\overrightarrow{\nabla} W}{W} \tag{64}$$

where W is the intrinsic wave impedance of the medium. The $\overrightarrow{v}$ and $\overrightarrow{W}$ are also expressed as

$$\overrightarrow{v} \quad := \quad \frac{\overrightarrow{\nabla}\sqrt{v}}{\sqrt{v}} \tag{65}$$

and

$$\overrightarrow{W} \quad := \quad \frac{\overrightarrow{\nabla}\sqrt{W}}{\sqrt{W}}. \tag{66}$$

Then equation (62) reduces to

$$M^{\overrightarrow{\varepsilon}}\overrightarrow{E} + ivM^{\overrightarrow{\mu}}\overrightarrow{B} \quad = \quad -(M^{\overrightarrow{v}}\psi + M^{\overrightarrow{W}}\psi*). \tag{67}$$

Then from equation (59), we obtain the generalized Dirac - Maxwell (GDM) equation of dyons for an inhomogenous medium as under,

$$\boxdot\psi + (M^{\overrightarrow{v}}\psi + M^{\overrightarrow{W}}\psi*) \quad = \quad iv\mu(-iv\overline{\rho} + j) = iv\mu j. \tag{68}$$

Equation (68) is completely analogous to the generalized Dirac - Maxwell (GDM) equation of dyons given by equation (42) and represents the quaternionoc form of Maxwell's equation for dyons in inhomogenous medium.

The time harmonic electromagnetic field has already defined by the equation (20) and (21) assuming that the sources are also time harmonic, i.e. [19],

$$\rho(x,t) = Re(\rho(x)e^{-i\omega t}) \tag{69}$$

and

$$\overrightarrow{j}(x,t) = Re(\overrightarrow{j}(x)e^{-i\omega t}). \tag{70}$$

As such, when electromagnetic field is considered to be monochromatic, we may obtain the following sets of quaternionic equations for generalized fields of dyons on substituting equations (20,21,69,70) into the equations (53) and (54) i.e.

$$D_{\overrightarrow{\varepsilon}}\overrightarrow{E} = i\alpha v\overrightarrow{B} - \frac{\rho_e}{\varepsilon} - \frac{\overrightarrow{j_m}}{\varepsilon} \tag{71}$$

and

$$D_{\overrightarrow{\mu}}\overrightarrow{B} = -\frac{i\alpha}{v}\overrightarrow{E} - \mu\rho_m + \mu\overrightarrow{j_e}. \tag{72}$$

In equation (71) and (72) the quantity $\alpha = \frac{\omega}{v}$ is denoted as the wave number. Hence in the absence of source equations (71) and (72) are reduces to

$$D_{\overrightarrow{\varepsilon}}\overrightarrow{E} = i\alpha v\overrightarrow{B} \tag{73}$$

and

$$D_{\overrightarrow{\mu}}\overrightarrow{B} = -\frac{i\alpha}{v}\overrightarrow{E}. \tag{74}$$

The medium is said to be slowly changing when its properties change appreciably over distances much greater than the wavelength [27]. It is described as the possibilty of reducing the generalized Dirac - Maxwell (GDM) equations of dyons (73) and (74) to the following Helmholtz equations,

$$(\triangle + \alpha^2)\overrightarrow{E} = 0 \tag{75}$$

and

$$(\triangle + \alpha^2)\overrightarrow{B} = 0 \tag{76}$$

where

$$\triangle + \alpha^2 = -(D+\alpha)(D-\alpha) = -D_\alpha D_{-\alpha}. \quad (77)$$

For checking the reduction , we consider that $|\overrightarrow{\varepsilon}|$ and $|\overrightarrow{\mu}|$ are very small and the terms containing the vectors $\overrightarrow{\varepsilon}$ and $\overrightarrow{\mu}$are supposed to be negligible. Then equations (73) and (74) takes the forms,

$$D\overrightarrow{E} = i\alpha v\overrightarrow{B} \quad (78)$$

and

$$D\overrightarrow{B} = -\frac{i\alpha}{v}\overrightarrow{E}. \quad (79)$$

Equations (78) and (79) can be diagonalized for the functions $\overrightarrow{\psi}$and $\overrightarrow{\psi}*$. As such, we obtain the quaternionic field equations in the following compact and consistent manner i.e.

$$D_{-\alpha}\overrightarrow{\psi} = 0 \quad (80)$$

and

$$D_\alpha\overrightarrow{\psi}* = 0. \quad (81)$$

5 Conclusion

In section - 3, we have used the Fourier transformation where any electromagnetic field is represented as an infinite superposition of time harmonic (monochromatic) fields. These fields are normally the main objects of study in radio electronics, wave propagation theory and many other branches of physics and engineering. Thus equations (20) and (21) describe the time harmonic electromagnetic field. As such, we have obtained the realization of the generalized Dirac - Maxwell's equation for the time varying fields and accordingly we get the physical laws given by equation (22). Equations (27) and (28) are the quaternionic differential equations analogous to time harmonic GDM equations given by (22). We have introduced new parameters for electric and magnetic field of dyons in the quaternionic form given by equation (29) and (30). Continuity equation for electric and magnetic charge in (isotropic) homogeneous medium is thus denoted by equations (31) and (32). As such, we have obtained the two decoupled equations (34) and (35) for the unknown vectors and (instead of electric and magnetic fields) which simplifies the analysis of the generalized Dirac - Maxwell's equation of dyons in homogeneous (isotropic) medium which can be diagonalized by (36) in the matrix form with the help of elementary transformations.

In section - 4, we have described the generalized electromagnetic fields of dyons in inhomogeneous media. In inhomogeneous media and is local depending on space and time coordinates as given by equation (39). Equations (40) are thus described as the generalized Maxwell's equation for dyons in isotropic inhomogeneous medium. The difference between the homogeneous and inhomogeneous media is that in former case the electric and magnetic parameter respectively (permittivity) and (permeability) are space - time independent while in the later case these are space-time dependent. As such, the differential form of Maxwell's equation in inhomogeneous medium is discussed by equations (43 - 46) while those in quaternionic form are described by the pair of equations (48) and (49). The new set of Maxwell's equation of dyon in inhomogeneous medium are discussed by equation (53) and (54). On the other hand, the quaternionic form of field equation of dyons in inhomogeneous media is described by equation (59). The electric and magnetic field are decomposed in terms of the complex vector field given by equation (60) and (61). As such, we may obtain equation (68q) as a generalization of the well known complex analysis Vekua equation describing generalized analytic functions [27]. This equation is also equivalent to the generalized Dirac - Maxwell equation of dyons given earlier by equation (1) and is analogous to the quaternionic form of Maxwell's equation of dyons in inhomogeneous medium. When the electromagnetic field is described monochromatic, we have discussed pair of GDM equations given by equation (71) and (72) in terms of wave number depending on wavelength. For source less field these equations are reduced accordingly to equation (73) and (74). It has been claimed that the medium is described as slowly changing when its properties are changed over distance much greater than the wavelength. We have thus obtained the wave equation given by equations (75 - 77) which reduces to equation (78) and (79).

Hence we may conclude that the presence of these results in the rotation of electromagnetic fields and its observable, particular in the microwave range of a particle consisting electric and magnetic charges (i.e. a dyon). Such experimental observations may be used in physical chemistry to characterize molecular structure. The present theory of generalized electrodynamics of dyons leads the connection between the mechanical parameters with the dielectric properties of the brain tissue considered as bio plasma. Hence the proposal for dyonic bio plasma plays an important role in the understanding of monopole and dyons. Our theory reduces to the theories described earlier [19, 20, 21, 22, 23, 24] for the case of electric charge in the absence of magnetic monopoles on dyon and consequently the theories of pure monopole be described from duality in the absence of electric charge on dyons. The quaternionic reformulation of generalized Maxwell's equations for the time - dependent electromagnetic fields here opens new window for various applications of quaternion analysis of the present theory.

References

[1] Dirac, P. A .M, Proc. R. Soc. London Sec , **A 133** (1931),60.

[2] Saha M. N., Phys. Rev., **75** (1949) 1968.

[3] Malkus M. V. R., Phys. Rev., **83** (1951) 899.

[4] Goto E., Kolm H. M. and Ford K. W., Phys. Rev. **132** (1963) 387.

[5] Rosenbaum D., Phys. Rev., **147** (1966) 891.

[6] Zwanziger D.; Phys. Rev. , **B137** (1965) 647.

[7] Goldhaber A. S., Phys. Rev., **B140** (1965) 1407.

[8] Schwinger J., Science, **165** (1969) 757.

[9] Goldhaber A. S., Phys. Rev. Lett., **36** (1976) 1122.

[10] Zwanziger D., Phys. Rev.,**176** (1968) 1489.

[11] Seo K., Phys. Lett., **B126** (1983) 201.

[12] Faddeev L. D., Les Houches lecture, Session 20, North Holland, (1976).

[13] Coleman S., Lectures delivered at the Int. school of Subnuclear Phys, (1975).

[14] Zwanziger D., Phys. Rev., **176** (1968) 1480.

[15] Cabibbo N. and Ferrari E., Nuovo Cim., **23** (1962) 1147.

[16] t' Hooft G. , Nucl. Phys., **B79** (1974) 276; **B138** (1978) 1.

[2] Polyakov A. M., JETP Lett, **20** (1974) 194.

[18] Julia B. and Zee A., Phys Rev , **D21** (1980) 2940.

[19] Kravchenko, V.V, Applied Quaternionic Analysis, Research and Exposition in Mathematics, **28** (2003), Heldermann Verlage, Germany.

[20] Kravchenko, V .V, Zeitschrift, Fur Analysis und Anwenduhgen **21** (2002) 21.

[21] Khmelnytskaya, K .V and Kravchenko, V. V, Zeitschrift Fur Analysis und Anwenduhgen, **56** (2001) 4641.

[22] Grudsky, S. M, K. V. Khmelnytskaya and Kravchenko, V. V., J. Phys. A. Math. Gen., **37** (2004) 4641.

[23] Singh Jivan, Bisht, P. S. and Negi, O. P. S ,Generalized fields of dyons in isotropic medium .arXiv:hep-th/0611208; Communication in Physics, (2007) (To be published).

[24] Singh Jivan, Bisht, P. S. and Negi, O. P. S, Quaternion analysis of generalized fields of dyons in isotropic medium. arXiv: hep- th/07033083; J. Phys. A: Math. and Gen.,

[25] Callen, C. G., Phys. Rev., **D25** (1982), 2141.

[26] Kravchenko, V. V. and Marco, P. Ramfrez, " A quaternionic reformulation of Maxwell' s equations for inhomogeneous media", Proceeding of the IEEE Guanajuato Mexico Section 1st International workshop in Mathematical modelling of physical processes inhomogeneous media, March 20 - 2, 2001 Guanjuato Mexico, 59 - 61.

[27] Babich, V. M. and Buldyrev, V. S., "Short wavelength diffraction theory asymptotic methods", Springer, Berlin (1991).