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NUMERICAL SOLUTIONS OF UNSTABLE DIRECT AND INVERSE DYNAMIC PROBLEMS

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A solution to a system of equations for a homogeneous space in the time and frequency domains for the case of a point source is obtained. Ray representations of waves of various types in an inhomogeneous elastic-porous medium are obtained. Inverse problems of determining the parameters of the medium and seismic moment tensor are considered. This is done by using the information about: 1) component parts of the displacement vectors of P - and S - waves at a fixed point of space; 2) measured at six fixed points of space for all times. The noise stability of the solutions to the inverse problems considered is investigated numerically with the use of the method of critical components.

Keywords: *Dynamic problems, Method of critical component, Inverse problem, Equation's system, Numerical solutions.*

INTRODUCTION

In many applied objects with prediction of earthquake [1], there is represented useful to develop mathematical apparatus of direct and inverse dynamic problems for equation's system, which is capable to describe a condition of display of such events. On an example of features of spreading elastic waves, which are caused by point source of concentrated force type, we shall show definition opportunity of event place, force and other characteristics.

It is known that mathematical models of wave propagation theory give an instrument to determine the numerical values of the propagation speeds and absorption coefficients of elastic waves. The more realistic and reliable the mathematical model, the more accurate the determined values of propagation speed and the absorption coefficient of elastic waves [2]. The peculiarities in present tense of elastic wave absorption in media that have been established by now and the simultaneous manifestation of multiple electro-seismic effects are not consistent with the simplest models [2, 3]. In the process of propagation, seismic waves dissipation, which is associated which energy absorption.

We obtain a solution to the linearized system of equations for the entire homogeneous space in the frequency domain in the case when wave processes are caused by a point source of the concentrated force type. To determine the seismic moment tensor, we use, in contrast to [4], a part of the components of the displacement vectors P-longitudinal and S- transverse waves measured only at one point source of the concentrated force type. The noise stability of the solutions to the inverse problems considered is investigated numerically with the use of the method of critical components.

DIRECT DYNAMIC PROBLEM

A nonlinear mathematical model of filtration is constructed in [2]. Linearized system of equations for the homogeneous medium has the following form [3, 6]:

$$\frac{\partial^2 \mathbf{U}}{\partial t^2} - c_s^2 \Delta \mathbf{U} - (a_1 - c_s^2)\nabla div\mathbf{U} + a_2 \nabla div \mathbf{V} = \mathbf{f}, \quad (1)$$

$$\frac{\partial^2 \mathbf{V}}{\partial t^2} - a_4 \nabla div\mathbf{V} + a_3 \nabla div \mathbf{U} = \mathbf{f}. \quad (2)$$

Here, $\mathbf{U} = (U_1, U_2, U_3)$ and $\mathbf{V} = (V_1, V_2, V_3)$ are the displacement vector of particles of the elastic porous body and liquid with the partial densities ρ_s and ρ_l, respectively, and $\mathbf{f}$ is the body force.

The main difference of linearized model from the well-known Frenkel-Biot models [6-8] is that it is described by three constants [2]. The coefficients $a_k,\ k = \overline{1,4}$ are expressed in terms of the speeds $c_s,\ c_{p_m}\ (m = 1, 2)$, as well as the ratios between the partial density ρ_l to the partial density ρ_s and look like as [3]

$$a_1 = \frac{\rho_l}{\rho}(c_{p_1}^2 + c_{p_2}^2) + \frac{4\rho_s^2}{3\rho^2}c_s^2 + \frac{\rho_s - \rho_l}{\rho}\tilde{z}, \quad a_2 = \frac{\rho_l}{\rho}(c_{p_1}^2 + c_{p_2}^2 - 2\tilde{z} - \frac{4}{3}c_s^2),$$

$$a_3 = \frac{\rho_s}{\rho}(c_{p_1}^2 + c_{p_2}^2 - 2\tilde{z} - \frac{4}{3}c_s^2), \quad a_4 = \frac{\rho_s}{\rho}(c_{p_1}^2 + c_{p_2}^2 - \frac{4}{3}c_s^2) + \frac{\rho_s - \rho_l}{\rho}\tilde{z},$$

$$\widetilde{z} = \frac{1}{2}(c_{p_1}^2 + c_{p_2}^2 - \frac{8\rho_s}{3\rho}c_s^2) + \sqrt{\frac{1}{4}(c_{p_1}^2 - c_{p_2}^2)^2 - \frac{16}{9}\frac{\rho_l \rho_s}{\rho^2}c_s^4}, \ \rho = \rho_s + \rho_l .$$

In the construction of the solution to the direct problems, we consider a source of point type, that is,

$$f_v(t, \mathbf{x}) = M_{v\beta}(t)\frac{\partial \delta(\mathbf{x} - \mathbf{y})}{\partial x_\beta}. \tag{3}$$

Here, $M_{\alpha\beta}(t) = M_{\beta\alpha}(t)$ is the seismic moment tensor [9], $\delta(t)$ is the Dirac function, $\mathbf{x} = (x_1, x_2, x_3)$ is the coordinate of the receiver, and $\mathbf{y} = (y_1, y_2, y_3)$ is the coordinate of the source.

The solution to equations (1), (2) with the right-hand side (3) in the frequency domain has the following form:

$$\hat{\mathbf{U}}_\alpha(\omega, \mathbf{x}) = \hat{\mathbf{U}}_\alpha^s(\omega, \mathbf{x}) - \frac{a_4 - c_{p_1}^2}{a_3}\frac{c_{p_2}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{U}}_\alpha^{p_1}(\omega, \mathbf{x})$$
$$+ \frac{a_4 - c_{p_2}^2}{a_3}\frac{c_{p_1}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{V}}_\alpha^{p_2}(\omega, \mathbf{x}), \tag{4}$$

$$\hat{\mathbf{V}}_\alpha(\omega, \mathbf{x}) = -\frac{c_{p_2}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{U}}_\alpha^{p_1}(\omega, \mathbf{x}) + \frac{c_{p_1}^2 - a_4 + a_3}{c_{p_1}^2 - c_{p_2}^2}\hat{\mathbf{V}}_\alpha^{p_2}(\omega, \mathbf{x}), \tag{5}$$

$$\hat{\mathbf{U}}_\alpha^s(\omega, \mathbf{x}) = \Delta\frac{\partial \hat{\mu}_{\alpha\beta}(\omega, c_{p_1}, \mathbf{x})}{\partial x_\beta} - \frac{\partial^3 \hat{\mu}_{\alpha\beta}(\omega, c_{p_1}, \mathbf{x})}{\partial x_\alpha \partial x_\beta \partial x_v},$$

$$\hat{\mathbf{U}}_\alpha^{p_1}(\omega, \mathbf{x}) = \frac{\partial^3 \hat{\mu}_{v\beta}(\omega, c_{p_1}, \mathbf{x})}{\partial x_\alpha \partial x_\beta \partial x_v}, \ \hat{\mathbf{V}}_\alpha^{p_2}(\omega, \mathbf{x}) = \frac{\partial^3 \hat{\mu}_{v\beta}(\omega, c_{p_2}, \mathbf{x})}{\partial x_\alpha \partial x_\beta \partial x_v},$$

$$\hat{\mu}_{\alpha\beta}(\omega, c, \mathbf{x}) = \hat{M}_{\alpha\beta}(\omega)\frac{e^{-i\frac{\omega}{c}|\mathbf{x}-\mathbf{y}|}}{4\pi\omega^2|\mathbf{x}-\mathbf{y}|}, \ |\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$

INVERSE PROBLEM OF DETERMINING THE SEISMIC MOMENT TENSOR

Let the arrival times transverse waves at some points of space and the coordinates of a seismic event be known. With the use of this information, it is necessary to determine the seismic moment tensor. Mathematically, the problem is formulated as follows: it is necessary to find $M_{\alpha\beta}(t)$ with the help of the following information:

$$\mathbf{U}^{p_1}(t, \mathbf{x})\Big|_{\mathbf{x}=\mathbf{x}_0} = \mathbf{U}^{p_1}(t), \tag{6}$$

$$\mathbf{U}^{s}(t, \mathbf{x})\Big|_{\mathbf{x}=\mathbf{x}_0} = \mathbf{U}^{s}(t). \tag{7}$$

Since the solution to the system of equations (1) and (2) is given in the frequency domain, the solution to the inverse problem will also be realized in the frequency domain. For this, we perform the Fourier transform of the data (6) and (7). Assuming in (4) that $\mathbf{x} = \mathbf{x}_0$ and using the notation $\mathbf{Z} = (\hat{M}_{11}(\omega), \hat{M}_{22}(\omega), \hat{M}_{33}(\omega), \hat{M}_{12}(\omega), \hat{M}_{13}(\omega), \hat{M}_{23}(\omega))^T$ (where T is the transposition sign), we obtain.

$$\mathbf{AZ} = \mathbf{Y}. \tag{8}$$

Here, $\mathbf{Y} = \mathrm{Re}(\hat{\mathbf{U}}^{p_1}(\omega, \mathbf{x}_0), \hat{\mathbf{U}}^{s}(\omega, \mathbf{x}_0))^T$ and, by virtue of (8), the components of the matrix $\mathbf{A} = (A_{ij})_{6\times 6}$ are real.

Thus, the solution to the inverse problem is reduced to the solution of system of linear algebraic equations (5).

Let the pore pressures at some points $\mathbf{x}_k$, $k = \overline{1, 6}$, of space be known. By these measurements, it is required to determine the seismic moment tensor. Mathematically, the problem in terms of the Fourier transform is formulated as follows: it is necessary to find $\hat{M}_{\alpha\beta}(\omega)$ by using the information

$$\hat{P}(\omega, \mathbf{x}) = \Big|_{\mathbf{x}=\mathbf{x}_k} = \hat{P}_k(\omega),\ k = \overline{1,\ 6}\ ,$$

where $P = \rho\ div(a_3\mathbf{U} - a_4\mathbf{V})$, and the coordinates of the source are known.

Assuming in (4) and (5) that $\mathbf{x} = \mathbf{x}_k$, $k = \overline{1,\ 6}$ and taking into account the pore pressure definition for $\mathbf{Z}$, we obtain

$$\widetilde{\mathbf{A}}\mathbf{Z} = \widetilde{\mathbf{Y}}, \tag{9}$$

where $\widetilde{\mathbf{Y}} = \mathrm{Re}(\hat{P}_1(\omega), \hat{P}_2(\omega), \hat{P}_3(\omega), \hat{P}_4(\omega), \hat{P}_5(\omega), \hat{P}_6(\omega))^T$ and the components of the matrix $\widetilde{\mathbf{A}} = (\widetilde{A}_{i\,j})_{6\times 6}$ are real.

NUMERICAL SOLUTIONS OF SYSTEM

To solve ill-posed systems of linear algebraic equations (5) and (6), we use the critical components method [5]. In this method, the numerical solution of systems $\mathbf{BX} = \mathbf{Y}$ with ill-posed matrices is reduced to the problem of stable solution of the reduced systems. The ill-posed components of the solution $x_{l_\mathbf{k}}$ are considered separately, and do not participate in the recurrent processes of deriving any components of the solution. Therefore, these components are called critical. The critical component method allows us to numerically find numerically an approximate normal minimal solution $\mathbf{X}^+$, which provides a minimum of the discrepancy norm

$$(\mathbf{X}^+ = \mathbf{B}^+\mathbf{Y}) : \left\|\mathbf{BX}^+ - \mathbf{Y}\right\| = \inf_{\mathbf{X}\in\mathbf{X}_\mathbf{B}} \left\|\mathbf{BX} - \mathbf{Y}\right\|,\ \left\|\mathbf{X}^+\right\| = \inf_{\mathbf{X}\in\mathbf{X}_\mathbf{B}} \left\|\mathbf{X}\right\|,$$

where $\mathbf{X}_\mathbf{B}$ is the totality of all solutions of the system $\mathbf{BX} = \mathbf{Y}$, and the unique matrix $\mathbf{B}^+$, satisfies the conditions

$$\left\|\mathbf{B}^+\mathbf{B} - \mathbf{E}\right\| = \inf_{\widetilde{\mathbf{B}}^{-1}\in\Omega_\mathbf{B}} \left\|\widetilde{\mathbf{B}}^{-1}\mathbf{B} - \mathbf{E}\right\|,\ \left\|\mathbf{B}^+\right\| = \inf_{\widetilde{\mathbf{B}}^{-1}\in\Omega_\mathbf{B}} \left\|\widetilde{\mathbf{B}}^{-1}\right\|,\ \mathbf{B}^+\mathbf{B} = \mathbf{BB}^+ \cdot$$

Here $\mathbf{E}$ is a unit matrix and $\Omega_\mathbf{B}$ is the totality of all $\widetilde{\mathbf{B}}^{-1}$, which are “pseudoinverse” to $\mathbf{B}$.

Let seismic moment tensor $M_{\alpha\beta}(t)$ is given by [4]:

$$M_{\alpha\beta}(t) = a \cdot \delta(t) \cdot \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix},$$

where $a = 10^{19}\ dine \cdot cm \cdot s$. The parameters of the medium take the following values:

$c_{p_1} = 6.1 km/s,\ c_{p_2} = 0.6 km/s,\ c_{p_s} = 3.5 km/s,$

$\rho_s = \rho_s^f (1 - d_0),\ \rho_l = \rho_l^f d_0,\ \ \rho_s^f = 2.7 g/cm^3,\ \rho_l^f = 0.9 g/cm^3,$

$d_0 = 10\%, \omega = 10 Hz, \alpha' = \beta' = 1.$

In Tables 1 and 2 $\widetilde{\mathbf{Z}} = (\widetilde{Z}_1, \widetilde{Z}_2, \widetilde{Z}_3, \widetilde{Z}_4, \widetilde{Z}_5, \widetilde{Z}_6)^T$, approximate solution to system (8) and (9), respectively, are given below. The relative solution errors $\delta = \|\mathbf{Z} - \widetilde{\mathbf{Z}}\| / \|\mathbf{Z}\|$ at the corresponding relative perturbations of the right-hand sides $\varepsilon = \|\mathbf{Y} - \widetilde{\mathbf{Y}}\| / \|\mathbf{Y}\|$ are also given.

Table 1. Approximate solutions of system (8)

ε	$\widetilde{Z}_1$	$\widetilde{Z}_2$	$\widetilde{Z}_3$	$\widetilde{Z}_4$	$\widetilde{Z}_5$	$\widetilde{Z}_6$	δ
10%	1.112	2.211	3.295	4.393	5.396	6.656	9.8%
20%	1.213	2.412	3.594	4.792	5.887	7.261	19.8%
30%	1.314	2.613	3.894	5.192	6.378	7.867	29.7%
40%	1.415	2.814	4.193	5.591	6.868	8.472	39.7%

Table 2. Approximate solutions of system (9)

ε	$\tilde{Z}_1$	$\tilde{Z}_2$	$\tilde{Z}_3$	$\tilde{Z}_4$	$\tilde{Z}_5$	$\tilde{Z}_6$	δ
10%	0.918	2.144	3.555	4.500	5.269	6.712	11.3%
20%	1.002	2.339	3.878	4.909	5.748	7.323	21.0%
30%	1.085	2.534	4.202	5.318	6.227	7.933	30.9%

CONCLUSION

It is seen from Tables (1) and (2) that the error of reconstruction of components of the seismic moment's tensor is at the error level.

Let's note that results of the present work are used in nuclear physics methods for earthquake prediction. In particular, in neutron method for the definition of event place, force and other characteristics.

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الحلول الرقمية للمسائل الديناميكية غير المستقرة الأمامية والعكسية

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تم الحصول على حل لمنظومة معادلات الفضاء المتجانس بالنسبة لمتطلبات الزمن والتردد لحالة المصدر النقطى . كما تم الحصول على تمثيل شعاعي لمختلف أنواع الموجات لوسط غير متجانس ذى طبيعة مسامية – مرنة . وقد تم تناول المسائل العكسية لتعين البارامترات الخاصة بدفع الكمية الممتدة ("التنزور") المتوسط والزلزالى (السيزمى) ، ذلك باستخدام معلومات عن :

1- الأجزاء المكونة لمتجهات الإزاحة للموجات P ، S عند نقطة معينة.

2- المعلومات المقاسة عند ست (6) نقاط فى الفضاء طوال الوقت.

وقد تمت دراسة رقمية لثبات الضوضاء بالنسبة لحلول المسائل العكسية المتناولة باستخدام طريقة المكونات الحرجة.