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STEADY PLANE COUETTE FLOW WITH POROSITY IN THE PRESENCE OF A CENTRIFUGAL FIELD WITH A VELOCITY DEPENDENT COLLISION FREQUENCY

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The plane steady Couette gas flow with porosity is studied in the presence of a centrifugal field. A model kinetic equation of the BGK (Bhatanger-Gross-Krook) type is used to replace Boltzmann kinetic equation where the collision frequency is assumed velocity dependent. We apply the method of moments to calculate the different macroscopic properties of the gas such as the mean velocity, shear stress, coefficient of viscosity as functions of the distance between two plates and the centrifugal Mach-number. Some non-equilibrium thermodynamic properties of the system are investigated namely the entropy, entropy production and internal energy change.

INTRODUCTION

In the analysis of the gas centrifuge, it is necessary to calculate the density and flow profile of the gas as function of position within the device [1], Uranium enrichment is the best known application .The enrichment in a single centrifuge is very small, so a large number of machines have to be arranged in series to obtain a sufficient degree of enrichment. The Moments method is used to replace the Boltzmann equation for obtaining a suitable solution for any Knudsen number [2]. The speed of rotation is very high and gives rise to a very strong centrifugal force. Due to this force the gas is distributed in a very non-uniform manner between the outer wall and the central axis. Indeed, calculations indicate that whilst the high density region near the outer wall can be treated by continuum hydrodynamics, the region near the axis behaves like a rarefied gas. Such behavior calls for special methods of analysis based upon a kinetic theory description of the gas. The Boltzmann equation for gases is know to describe the properties of non-equilibrium states to a high degree of accuracy [3] and it should therefore, in principle tend to a precise assessment of the behavior of a gas in the centrifuge. Practical centrifuges are of complex internal shape although generally of cylindrical symmetry. However, even the problem of a rotating cylinder is not simple to solve without extensive numerical computation and so alternative methods have been suggested for gaining initial into this problem. Pomraning [4] has proposed a simple model of the centrifugal force in plane geometry using a fictitious force term in the linear Boltzmann equation to simulate the centrifugal effect. This equation was solved by Pomraning through the use of the half ranges expansion technique using orthogonal polynomials. It was pointed out by Johnson [3] that Pomraning's solution contains an error which leads to non-conservation of momentum. This error has been corrected by Johnson who has also provided an exact solution of the problem in the Knudsen limit. A full review of past work in the application of the kinetic theory to this problem can also be found in Johnson's papers [3,5]. Recently a series of works conducted by Aleksandrov [6-8] has been devoted to correct the ideas and optimize the applications of gas centrifuge. The advancements associated with modern day technology demands incorporation of the best physical understanding and the most accurate modeling of gas centrifuge in gases, so analytical or numerical solution of these model-based equations always requires the use of approximations, especially concerning the BGK model. Consequently, none of the existing methods of calculation are valid for an actual centrifuge, although they are valuable for understanding the physical phenomena that take place within the gas centrifuge. In recent times there have been major advances in the fundamentals, and this is the subject of the present paper.

THE PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

We consider in this paper the Pomraning problem plus porosity. A rarefied gas flows between two infinite porous parallel plates. We take the y-axis perpendicular to the plates and choose the origin of coordinates so that the plates are located in the plane $y = \pm d/2$ where d is the plates separation. Let the velocities of the plates be respectively $\pm u/2$ in the x-direction and the two plates are at equal constant temperature, an external force acts on the y direction of strength $\left(\frac{F}{m}\right) = \omega^2 y$ per unit mass, where ω is a constant of dimension rad/sec. To investigate the effect of the porosity of the plates, we consider that the gas flows out from the lower plate with velocity $V_1 = -au$ and from the upper plate with velocity $V_2 = bu$ respectively. Here a, b indicates the case of suction from the containing vessel.

The Boltzmann equation may be written in the following form [3]:

$$\left[\frac{\partial}{\partial t}+\overline{V}.\overline{\nabla}+\frac{\overline{F}}{m}.\frac{\partial}{\partial \overline{V}}\right] f = J[f] \ , \tag{1}$$

where $J[f\ ]$ is the usual Boltzmann collision operator, here f (r`, v`, t) is the distribution function dependent on the position r`, velocity v`, and time t, $\overline{V}$ is the molecular velocity and $\overline{F}$ is the external force acting on the gas. For the Pomraning problem with force given by $\overline{F}$ = mω^2y for the steady flow in the y- direction, so that Eq. (1) reduces to

$$\left[C_y\frac{\partial}{\partial y}+\omega^2 y\frac{\partial}{\partial C_y}\right] f = J[f]. \tag{2}$$

A model based on the Boltzmann equation with a velocity-dependent collision frequency [9] is suggested. The collision term in Eq. (2) can be written in the B.G.K model [10] as *J[f]* = -*ν*(*f*$_o$ – *f*), where ν is the collision frequency that depends on the square of the velocity, it is proportional to the number density in the centrifugal field $n(y) = n_0 e^{[-M^{*2}(1-y^2)]}$, n_0 is at the walls where y = ± d/2 [5], so that we can right the collision term in the form

$$J[f] = -A(C_x^2 + C_y^2)e^{-M^{*2}(1-y^2)}(f_o - f),$$

where A is a constant of dimension sec/cm^2, $f = n\left(\frac{\alpha}{\pi}\right)^{3/2} e^{[-\alpha(\overline{C}-\overline{V})^2]}$, is the non-equilibrium distribution function while $f_0 = n\left(\frac{\alpha}{\pi}\right)^{3/2} e^{[-\alpha\overline{C}^2]}$ is the equilibrium distribution function, $\alpha = \frac{1}{2RT_0} = \frac{1}{V_T^{\ 2}}$, R is the universal gas constant , T_0 is the constant temperature and V_T is the thermal velocity. The collision term *J[f]* in Eq.(2) is treated by the Liu et al. approach [11], which approximates the distribution function itself. The equation governing the chosen N parameters are obtained by taking N moments of the Boltzmann equation. The Liu et al. [11] distribution function is written in two discontinuous parts on both sides (1,2) of the cone of influence [10] in such a way that for the plane Couette flow with porosity *f* is defined by:

$$f^{\pm} = f_1^{\pm} + f_2^{\pm}$$

where

$$f_{1,2}^{\pm} = n_o\left(\frac{\alpha}{\pi}\right)^{3/2}\left(1+2V_{1,2}\alpha C_y\right)e^{-\alpha\left((C_x - V_{x1,2}^{\pm})^2 + C_y^2 + C_z^2\right)}$$

the minus or plus sign describes downward or upward going particles. In this problem we want to investigate the variation of the mean velocity $V_x^{\pm}=\left(V_{x_1}^{\pm}+V_{x_2}^{\pm}\right)/2$, the component of the shear stress (pressure deviator) P_{xy} and the viscosity coefficient μ of the gas as functions of the distance y between the moving plates at different values of the force Mach number, and porosity parameters. Thus Eq. (2) yields

$$\left[C_y\frac{\partial}{\partial y}+\omega^2 y\frac{\partial}{\partial C_y}\right]f=-e^{-M^{*2}(1-y^2)}A(C_x^2+C_y^2)(f_o-f). \tag{3}$$

Now multiplying Eq. (3) by any weighting function of the velocity $\theta_I(C)$ and integrating with respect to $d\bar{C}$ [12,13], in order to get the moment equations suitable for our purpose, therefore

$$\frac{d}{dy}\left[\int c_y f\,\theta_I d\bar{c}\right]-\omega^2 y\int f\,\frac{d\theta_I}{dc_y}d\bar{c}=Ae^{-M^{*2}(1-y^2)}\int\left[(C_x^2+C_y^2)\theta_I(f_{.}-f_o)\right]d\bar{c}\,. \tag{4}$$

Two equations are needed

(i) By taking $\theta_I = 1$ in Eq. (4)

$$\frac{d}{dy}\left[\int^{\infty}\int_{-\infty}^{o}\int^{\infty}c_y f^-\,d\bar{c}+\int_{-\infty}\int_{o}^{\infty}\int_{-\infty}c_y\,f^+d\bar{c}\right]=$$

$$Ae^{-M^{*2}(1-y^2)}\left[\int^{\infty}\int_{-\infty}^{o}\int^{\infty}\left(c_x^2+c_y^2\right)f^-\,d\bar{c}+\int_{-\infty}\int_{o}^{\infty}\int_{-\infty}(c_x^2+c_y^2)\,f^+\,d\bar{c}-\int\int_{-\infty}^{\infty}\int\ (c_x^2+c_y^2)\,f_o d\bar{c}\right],$$

Substituting the values of $f^{\pm}$ and integrating

$$\frac{d}{dy}\left[n\frac{V_1+V_2}{2}\right]=Ae^{-M^{*2}(1-y^2)}n\left[\frac{V_{x_1}^{\pm2}+V_{x_2}^{\pm2}}{2}+\frac{3(V_2-V_1)}{2\sqrt{\pi}\sqrt{\alpha}}+\sqrt{\frac{\alpha}{\pi}}\left(V_{x_2}^{\pm2}V_2-V_{x_1}^{\pm2}V_1\right)\right]. \tag{I}$$

(ii) Taking $\theta_I = C_y$ in Eq. (4)

$$\frac{d}{dy}\left[\int^{\infty}\int_{-\infty}^{o}\int^{\infty}c_y^2\ f^-\,d\bar{c}+\int_{-\infty}\int_{o}^{\infty}\int_{-\infty}c_y^2\ f^+d\bar{c}\right]-\omega^2 y\left[\int^{\infty}\int_{-\infty}^{o}\int^{\infty}\ f^-\,d\bar{c}+\int_{-\infty}\int_{o}^{\infty}\int_{-\infty}f^+d\bar{c}\right]$$

$$=Ae^{-M^{*2}(1-y^2)}\left[\int^{\infty}\int_{-\infty}^{o}\int^{\infty}c_y\left(c_x^2+c_y^2\right)f^-\,d\bar{c}+\int_{-\infty}\int_{o}^{\infty}\int_{-\infty}c_y(c_x^2+c_y^2)\,f^+\,d\bar{c}-\int\int_{-\infty}^{\infty}\int c_y(c_x^2+c_y^2)\,f_o d\bar{c}\right],$$

After integrating the above equation we get

$$\frac{d}{dy}\left[n\left(\frac{1}{2\alpha}+\frac{(V_2+V_1)}{\sqrt{\alpha\pi}}\right)\right]-\omega^2 y\, n\left[1+\frac{\sqrt{\alpha}}{\sqrt{\pi}}(V_2-V_1)\right]$$
$$=Ae^{-M^{*2}(1-y^2)}n\left[\frac{V_{x_2}^{\pm 2}+V_{x_1}^{\pm 2}}{2\sqrt{\alpha\pi}}+\frac{V_1-V_2}{\alpha}+\frac{1}{2}\left(V_{x_1}^{\pm 2}V_1-V_{x_2}^{\pm 2}V_2\right)\right]. \qquad [II]$$

All the variables and parameters of interest can be put in dimensionless form:

$\bar{V}_{x_{1,2}}=\alpha^{1/2}V_{x_{1,2}}$, $\bar{n}(y)=n(y)/n_o=e^{-M^{*2}(1-y^2)}$, d/dy = 2/d. $d/d\bar{y}$

V$_1$ =-$(M/\sqrt{\alpha})$a , V$_2$ =$(M/\sqrt{\alpha})$b

where M is the plate Mach number = $\alpha^{1/2}u$, M^* is the centrifugal force Mach number = ω $\sqrt{\alpha}$ d/2, both a,b are real numbers.

Hence Eqs. (I), (II) can be rewritten after removing the primes as:

$$\frac{\mathrm{d}}{\mathrm{dy}}\left[\mathrm{e}^{-\mathrm{M}^{*2}(1-\mathrm{y}^2)}\mathrm{M}\frac{(\mathrm{b}-\mathrm{a})}{2}\right]=\bar{\mathrm{A}}\,\mathrm{e}^{-2\mathrm{M}^{*2}(1-\mathrm{y}^2)}\left[\frac{\mathrm{V}_{\mathrm{x}_1}^{\pm 2}+\mathrm{V}_{\mathrm{x}_2}^{\pm 2}}{2}+\frac{3\mathrm{M}(\mathrm{b}+\mathrm{a})}{2\sqrt{\pi}}+\frac{\mathrm{M}(\mathrm{a}\mathrm{V}_{\mathrm{x}_1}^{\pm 2}+\mathrm{b}\mathrm{V}_{\mathrm{x}_2}^{\pm 2}}{\sqrt{\pi}}\right], \qquad (5)$$

$$\frac{d}{dy}\left[e^{-M^{*2}(1-y^2)}\left(1+\frac{M(b+a)}{\sqrt{\pi}}\right)\right]-M^{*}ye^{-M^{*2}(1-y^2)}\left(1+\frac{M(b+a)}{\sqrt{\pi}}\right)$$
$$=\bar{A}e^{-2M^{*2}(1-y^2)}\left[\frac{V_{x_2}^{\pm 2}-V_{x_1}^{\pm 2}}{2\sqrt{\pi}}+M(b-a)+M\,\frac{(bV_{x_2}^{\pm 2}-bV_{x_1}^{\pm 2}}{2}\right]. \qquad (6)$$

where $\bar{A}=\frac{A}{\sqrt{\alpha}}.d/2$.

Differentiation of Eq. (5) w.r.t. y yields

$$\frac{M^{*2}yM}{\bar{A}}(b-a)e^{+M^{*2}(1-y^2)}=V_{x_2}^{\pm 2}\left(1/2+\frac{Mb}{\sqrt{\pi}}\right)+V_{x_1}^{\pm 2}\left(1/2+\frac{Ma}{\sqrt{\pi}}\right)+\frac{3M(b+a)}{2\sqrt{\pi}}$$

$$\frac{M^{*2}yM}{\bar{A}}(b-a)e^{+M^{*2}(1-y^2)}=B^{*}V_{x_2}^{\pm 2}+K^{*}V_{x1}^{\pm 2}+\frac{3}{2}\frac{M}{\sqrt{\pi}}(b+a)\,. \qquad (7)$$

Similarly Eq. (6) yields

$$\frac{2M^{*2}y}{\bar{A}}\left[\sqrt{\pi}+M(b+a)\right]e^{M^{*2}(1-y^2)}=BV_{x_2}^{\pm 2}-KV_{x_1}^{\pm 2}+2\sqrt{\pi}M(b-a)\,, \tag{8}$$

where $B = 1+\sqrt{\pi}Mb$, $B^{*} = \frac{1}{2}+\frac{bM}{\sqrt{\pi}}$

$K = 1+\sqrt{\pi}Ma$, $K^{*} = \frac{1}{2}+\frac{aM}{\sqrt{\pi}}$

Solving Eqs. (7), (8) together we get

$$V_{x_2}^{\pm 2}=\frac{yM^{*2}e^{M^{*2}(1-y^2)}}{S\bar{A}}L_1-\frac{2}{S}K^{*}\sqrt{\pi}M(b-a)-\frac{3KM}{2S\sqrt{\pi}}(b+a)\,, \tag{9}$$

$$V_{x_1}^{\pm 2}=\frac{yM^{*2}e^{M^{*2}(1-y^2)}}{S\bar{A}}L_2+\frac{2}{S}B^{*}\sqrt{\pi}M(b-a)-\frac{3BM}{2S}(b+a)\,,$$

where $S = K^{*}B + B^{*}K$ is the suction factor, and $L_1 = 2\mathrm{K}^{*}\left(\sqrt{\pi}+\mathrm{M(b+a)}\right)+\mathrm{KM(b-a)}$, $L_2 = \left[\mathrm{BM(b-a)}-2\mathrm{B}^{*}(\sqrt{\pi}+\mathrm{M(b+a)}\right]$

The mean velocity can be defined in a simple form

$$V_x^{\pm}=\frac{\left(V_{x_2}^{\pm}+V_{x_1}^{\pm}\right)}{2} \tag{10}$$

The pressure deviator is calculated from the integral

$$P_{xy}^{\pm} = \int c_x c_y f^{\pm} d\bar{c} = \int_{-\infty}^{\infty}\int^{o}\int^{\infty} c_x c_y f^{-}\, d\bar{c}+\int_{\infty}\int_{o}^{\infty}\int_{-\infty} c_y c_x\, f^{+} d\bar{c}$$

$$=\left[{}^{1}\!/_{2}\left(V_1V_{x_1}^{\pm}+V_2V_{x_2}^{\pm}\right)+\frac{1}{2\sqrt{\pi}\sqrt{\alpha}}\left(V_{x_2}^{\pm}-V_{x_1}^{\pm}\right)\right]$$

or in dimensionless form

$$P_{xy}^{\pm}=\frac{e^{-M^{*2}(1-y^2)}}{2\sqrt{\pi}}\left[BV_{x_2}^{\pm}-K\;V_{x_1}^{\pm}\right] \tag{11}$$

The viscosity coefficient μ is defined as

$$\mu = -P_{xy}\left(\frac{\partial V_x(y)}{\partial y}\right)^{-1}\,, \tag{12}$$

where $$\frac{dV_x}{dy}=\frac{(M^{*2}-2M^{*4}y^2)\,e^{M^{*2(1-y^2)}}}{S\bar{A}}\left[\frac{L_1V^{\pm}_{x_2}+L_2V^{\pm}_{x_1}}{4\sqrt{V^{\pm}_{x_1}}\,\sqrt{V^{\pm}_{x_2}}}\right]$$

THE THERMODYNAMICAL PROPERTIES OF THE SYSTEM

The entropy:

We begin with the evaluation of the entropy per unit mass $\bar{s}$ [14 -16]. It is written in dimensionless form as:

$$\begin{aligned}\bar{s}&=-\int f\ln f\,d\bar{c}=-\left(\int f_1\ln f_1 d\bar{c}+\int f_2\ln f_2 d\bar{c}\right)\\ &=-e^{-M^{*2}(1-y^2)}[\{-M^{*2}(1-y^2)+1\}(1+\frac{M}{\sqrt{\pi}}(b+a))\\ &-\frac{M}{\sqrt{\pi}}(a+b)+M^2(b^2+a^2)-\frac{3}{2}]\end{aligned} \tag{13}$$

The entropy production:

The Boltzmann’s local entropy production inequality has the form [17-20]

$$\sigma=-k\int J(f,f^0)\ln f d\bar{c}\geq 0\,.$$

After substitution of the collision term it reads

$$\sigma=-kAe^{-M^{*2}(1-y^2)}\left(\int\left(c_x^2+c_y^2\right)f\ \ln f d\bar{c}-\int(c_x^2+c_y^2)f_o\ln f d\bar{c}\right)$$

Therefore after performing the integrals the entropy production in dimensionless form reads

$$\begin{aligned}\sigma=e^{-2M^{*2}(1-y^2)}&\left[\left(M^{*2}(1-y^2)\right)\left(\frac{V_{x_1}^{\pm 2}+V_{x_2}^{\pm 2}}{2}+\frac{3M(b+a)}{2\sqrt{\pi}}+\frac{M(bV_{x_2}^{2}+aV_{x_1}^{2})}{\sqrt{\pi}}\right)\right.\\ &\left.-\frac{3\pi M(b+a)}{2}+\frac{9M(b+a)}{2\sqrt{\pi}}-2M^2(b^2+a^2)-M^2(b^2V^{\pm 2}_{x_2}+a^2V^{\pm 2}_{x_1})-\frac{V^{\pm 2}_{x_2}+V^{\pm 2}_{x_1}}{4}\right]\end{aligned} \tag{14}$$

The internal energy change ***dU*** of the substance according to the first and second laws of thermodynamics is considered a function of the temperature T and volume V:

$$dU=T\,dS-P\,dV\ , \tag{15}$$

where $dV=-(dn/n^2)$ and $P=T\dfrac{\partial S}{\partial V}$.

DISCUSSION AND CONCLUSIONS

In this work we have studied the plane Couette flow problem with porosity in the presence of a centrifugal field. From the evaluation of the formulas (10-15), we obtain the following results illustrated in Figures 1.-12. where we take the plates Mach number $M=10^{-2}$

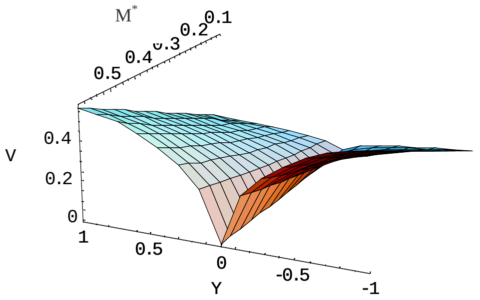

Figure 1. The mean velocity V vs. M^* and Y at a=b=0

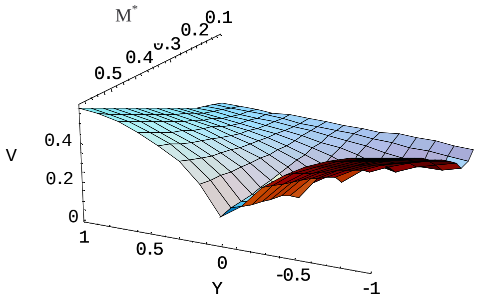

Figure 2. The mean velocity V vs. M^* and Y at a=b=2.5

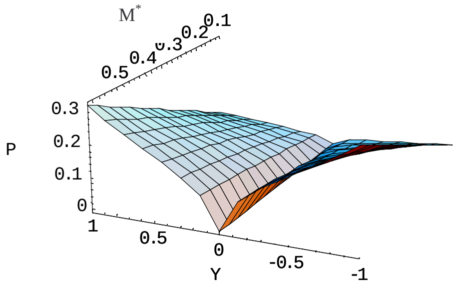

Figure 3. The shear stress P vs. M^* and Y at a=b=0

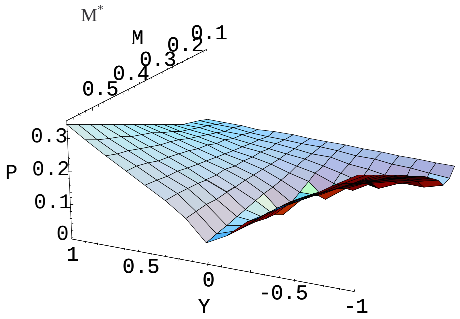

Figure 4. The shear stress P vs. M^* and Y at a=b=2.5

It is concluded that:

1) The flow velocity of the gas increases nonlinearly as the Mach number of the centrifugal force M^* increases beginning from zero in the absence of porosity a=b=0 at the central axis towards the plates, see Figure 1., which agrees with the result obtained by Johnson [21,22]. As the effect of the porosity takes place, a=b=2.5, making the flow velocity nonzero at the central axis increases, as shown in Figure 2.

2) The pressure deviator behaves in the same manner as the flow velocity. It does not change significantly with any values of the Mach number. This could be reached

experimentally by keeping the concentration of the working gas, isotopes of UF_6, constant inside the vessel [23], see in Figures 3., 4.

3) The viscosity coefficient decreases as the centrifugal Mach number increases, see in Figures 5., 6.

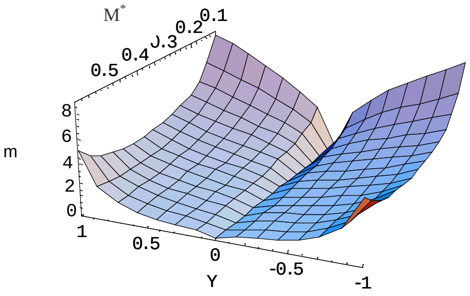

Figure 5. The viscosity coefficient μ vs. M^* and Y at a=b=0

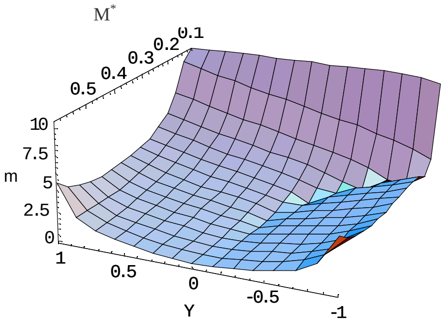

Figure 6. The viscosity coefficient μ vs. M^* and Y at a=b=2.5

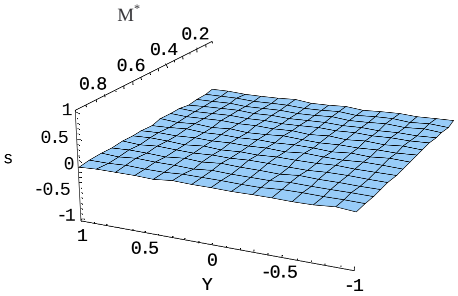

Figure 7. The entropy production σ vs. M^* and Y at a=b=0

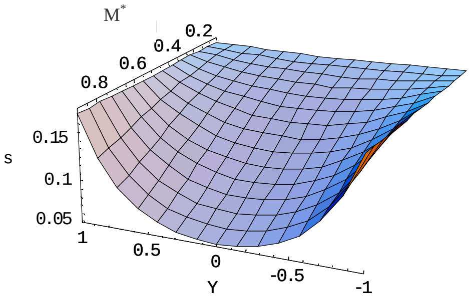

Figure 8. The entropy production σ vs. M^* and Y at a=b=2.5

4) The thermodynamics characteristics

I) Through the choice of the controlling parameters in the system it is revealed that the entropy production σ is always a positive quantity when the porosity is nonzero, see Figures 8., and reaches a minimum value at the center i.e. the equilibrium state, which is a good agreement with the Boltzmann H-theorem [14-16,21,22], σ vanishes in the absence of porosity, see Figure 7., and reaches a minimum value at the center i.e. the equilibrium state.

II) The entropy s equals zero at the small values of the centrifugal Mach number. It increases nonlinearly as the Mach number M^* increases to reach a maximum value i.e. the equilibrium state at the center. It shown that the behavior of the entropy is not affected when the two plates have equal porosity, see Figures 9., 10.

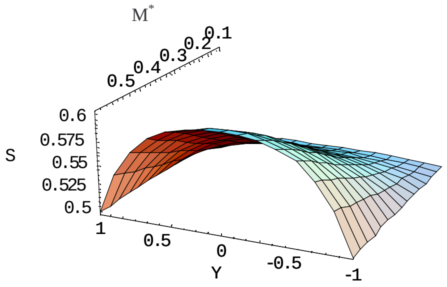

Figure 9. The entropy S vs. M^* and Y at a=b=0

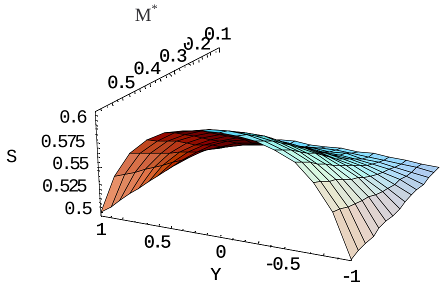

Figure 10. The entropy S vs. M^* and Y at a=b=2.5

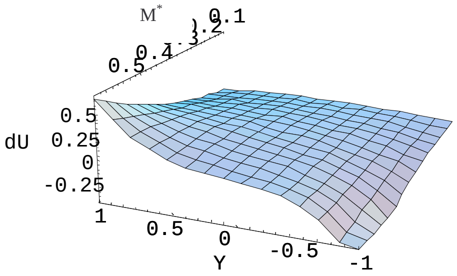

Figure 11. The internal energy change dU vs. M^* and Y at a=b=0

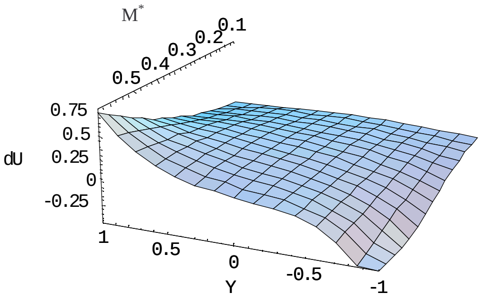

Figure 12. The internal energy change dU vs. M^* and Y at a=b=2.5

III) The internal energy change dU, see Figures 11., 12., reaches a maximum value at the upper plate when the porosity a=b=2.5 and is suffering a maximum loss at the lower plate. As a consequence of energy conservation it is shown that the energy loss at the lower space is compensated at the upper space, dU increases as the Mach number of the centrifugal force increases.

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سريان كويت المستوى غير المعتمد على الزمن مصحوبا بالمسامية في وجود مجال طرد مركزي واعتماد تردد التصادمات على السرعة

على ماهر عبدالخالق أبورابية و فاطمة محمد سالم المالكى*

كلية العلوم – جامعة المنوفية – شبين الكوم – مصر

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تمت دراسة سريان كويت المستوى غير المعتمد على الزمن مصحوبا بالمسامية في وجود مجال طرد مركزي واعتماد تردد التصادمات على السرعة استعانة بمعادلة بهاتناجر– جروس – كروك (B-G-K) لتحل محل معادلة بولتزمان أخذا في الاعتبار اعتماد تردد التصادم على السرعة. مع تطبيق طريقة العزوم أمكن حساب خواص عيانية للغاز مثل السرعة المتوسطة والإجهاد القصي ومعامل اللزوجة كدوال في المسافة بين اللوحين وعدد ماخ للطرد المركزي . تم إيجاد بعض الخواص الديناميكية الحرارية بعيده عن الاتزان للمنظومة وبصفة خاصة الانتروبيا ومعدل إنتاج الانتروبيا والتغير في الطاقة الداخلية.