

OPTIMIZATION APPROACH FOR SHIELDING COST OF MEDICAL CYCLOTRONS WITH FUZZINESS

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Most radiation fields are combinations of different kinds of radiation. The radiations of most significance are fast neutrons, thermal neutrons, primary gammas and secondary gammas. Thermo's composite shielding materials are designed to attenuate these types of radiation. The shielding design requires an accurate cost-benefit analysis based on uncertainty optimization technique. The theory of fuzzy sets has been employed to formulate and solve the problem of cost-benefit analysis of medical cyclotron. This medical radioisotope production cyclotron is based in Sydney, Australia.

Keywords: *Shielding Cost, Optimization, Fuzzy Parameter.*

INTRODUCTION

Cyclotron is a device that accelerates charged particles to high speeds by making them move faster and faster in a circular path. It is a primary source for the creation of radioactive materials used in the medical field as diagnostic tracers and for cancer treatment. The use of materials protecting from radiation is called shielding. So that effective radiation shielding is imperative to save operation of modern cyclotrons producing large activities of short-lived radioisotopes on a commercial basis. The use of lead or concrete around a source of radiation is preventing the harmful release of radiation.

The optimized shielding design of commercial medical cyclotrons producing large yields of medical radioisotopes requires a cost-benefit analysis of the radiological and

economical parameters [1]. The cost of radiological protection of a cyclotron facility is a function of the intensity of radiation fields produced inside and outside the vault. Also it is a function of cyclotron operational conditions, cost of shielding material, real estate cost and the depreciation rate of the cyclotron facility. The aim of the cyclotron shielding design is to achieve an optimized balance between the costs of radiological protection and radiological health. The mathematical optimization method of the accelerator shielding design has been discussed elsewhere [2].

In this paper, we will present an optimization model for the radiation process for medical cyclotrons problem with fuzziness where the thickness of cyclotron shielding is characterized by a fuzzy parameter. In fact the fuzzy decision making which allows various different degree of control is ideally suited for such practical problems. In addition, this approach has the following advantages [3]:

1. The problem is simplified and the representation is more realistic and practical. This is because we are treating a fuzzy and not well defined problem as it is.
2. The use of fuzziness to represent the thickness of cyclotron shielding offers exceptional flexibility for the decisions proposed.

FUZZY CONCEPTS AND DECISIONS

Fuzzy sets were first advocated by L. A. Zadeh in 1965. Fuzzy theory is a mathematical theory, and what is called fuzziness takes in one aspect of uncertainty [4], is the ambiguity that can be found in the definition of a concept or the meaning of a word. Fuzzy set theory is developed for solving problems in which descriptions of activities and observations are imprecise, vague, and uncertain.

Definition 1. (Triangular fuzzy number). It is a fuzzy number M whose membership function $\mu_M(x)$ satisfies the following conditions

- (1) A continuous mapping from \mathbb{R} to the closed interval $[0,1]$,
- (2) $\mu_M(x) = 0 \forall x \in (-\infty, a]$,
- (3) Strictly increasing and continuous on $[a,m]$,
- (4) $\mu_M(x) = 1$ at $x = m$,
- (5) Strictly decreasing and continuous on $[m,b]$,
- (6) $\mu_M(x) = 0 \forall x \in [b, +\infty)$,

where a, b and m are real numbers, and the fuzzy number is denoted by (a, m, b) .

Definition 2. (α -cut). The α -cut of a fuzzy set A is defined as an ordinary set A_α for which the degree of its membership function exceeds the level α :

$$A_\alpha = \{x \mid \mu_M(x) \geq \alpha, \alpha \in [0,1]\}.$$

From the definition, the α -cut of the triangular fuzzy number (a, m, b) is

$$A_\alpha = [a + \alpha(m - a), b - \alpha(b - m)].$$

SHIELDING CALCULATIONS

The nature of the radiation field produced in the vicinity of the cyclotron target bombarded with protons should be well known prior to the shielding calculation of the target vault wall. During routine isotope production, a well focused proton beam (30 MeV, 400 μ A) is guided into the target vault through the beam tube. The neutron (H_N) and gamma (H_G) dose equivalent rates can be expressed as follows [1]:

$$H_N (\text{Svh}^{-1}\text{m}^2) = 1.4 \pm 12\%$$

$$H_G (\text{Svh}^{-1}\text{m}^2) = 0.11 \pm 11\%$$

The total dose equivalent (neutron and gamma) at the external surface of the concrete shielding is calculated as

$$H_t = 2.4I \exp((-x / \lambda) / (d + x)) \quad (1)$$

where (μ A) is the proton beam current, x (m) is thickness of the concrete shielding, d is the distance between the target and the internal surface of the vault wall and λ (m) is the effective neutron attenuation length in heavy concrete = 0.126 m.

The thermal neutron fluence ϕ_n (neutrons. $\text{cm}^{-2}\text{s}^{-1}$) in the target vault is given as [5]:

$$\phi_n = \langle c \rangle Q / S \quad (2)$$

where $\langle c \rangle$ = fluence correction coefficient = 4, Q (neutrons. s^{-1}) = total number of neutrons produced in the target per second, S (cm^2) = total internal surface area of the vault.

THE OPTIMIZATION CALCULATIONS

The main goal of the optimization calculation is to reduce the total cost, made of the cost of shielding and the cost of radiological health. The mathematical form of the cost function for this optimization problem is described as follows:

$$C = X + Y \quad (3)$$

where C is the total cost and it is depending on engineering parameter, cyclotron operational parameter and relevant monetary values. X is the cost of radiation protection including radiological shielding C_S , real estate C_E and radioactive waste management C_W . This cost is depending on the engineering parameter and relevant monetary values. Finally, Y is the cost of radiological health detriment and it is depending on cyclotron operational parameter and monetary relevant values [1].

Equation 3. is called the objective function and it is subject to the following constraint

$$H_e(t) \leq H_L \quad e = 1, 2, \dots, n \quad (4)$$

where $H_e(t)$ is the effective dose equivalent (mSv/year) delivered to e^{th} individual at contact with the shielding thickness and H_L is the permissible average collective dose equivalent limit (mSv/year) [6].

ESTIMATION OF THE COST OF RADIATION PROTECTION

The net volume (V) of the shielding concrete, total surface area (F) and the total internal wall surface area (S) of the target vault wall are calculated as:

$$V (m^3) = ((a+x).(b+x) - ab)h \quad (5)$$

$$F (m^2) = (a+x).(b+x) \quad (6)$$

$$S (m^2) = 2(ab + ah + bh) \quad (7)$$

where $a (m)$ = length of the target vault, $b (m)$ = breadth of the target vault,
 $h (m)$ = height of the target vault, x = shielding thickness.

The cost of shielding (C_S), real estate (C_E) and radioactive waste disposal (C_W) are calculated as follows:

$$C_S (\$) = Vs \quad (8)$$

By substituting the value of V from equation (5) in equation (8) one gets:

$$C_S (\$) = ((a+x).(b+x) - ab)hs \quad (9)$$

where s = cost of 1 m^3 of shielding concrete (\$).

$$C_E (\$) = Fl \quad (10)$$

By substituting the value of F from equation (6) in equation (10) one gets:

$$C_E (\$) = (a + x).(b + x)l \quad (11)$$

where l = real estate rate of 1 m² of floor space (\$).

$$C_W (\$) = A_{SAT} p \quad (12)$$

where A_{SAT} is the saturation activity of ⁵⁹Fe which is present in the structure steel of the target station. A_{SAT} (Bq) of ⁵⁹Fe is calculated as:

$$A_{SAT} (Bq) = N \phi_n \sigma \quad (13)$$

where N = total number of ⁵⁹Fe atoms in the steel structure, ϕ_n = neutron fluence rate, σ = cross section of the ⁵⁹Fe(n, γ) reaction = 1.15 x 10⁻²⁴ (cm²) [7]. The total number of ⁵⁹Fe atom (N) could be calculated from the weight of the steel structure and the isotopic abundance of ⁵⁹Fe.

$$N = (0.01KL) \cdot \left(\frac{W}{A_{Fe}} \right) \quad (14)$$

where K = isotopic abundance of ⁵⁸Fe = 0.28 %, L = Avogadro's number = 6.022x10²³ atom.mol⁻¹, A_{Fe} = Atomic weight of iron = 55.85.

The neutron fluence rate given in equation (2) could be written as follows:

$$\phi_n = \langle c \rangle qI / S \quad (15)$$

where q = neutron production rate of solid copper target = 6.4 x 10¹⁰ (neutron. μ A⁻¹) [8]. By substituting the values of N from equation (14), ϕ_n from equation (15) and the numerical values of K , $\langle c \rangle$, q , L , σ and A_{Fe} in equation (12), A_{SAT} (Bq) of ⁵⁹Fe was calculated as:

$$A_{SAT} (GBq) = 9.83IW / S \quad (16)$$

Again, by substituting the values of A_{SAT} from equation (16) and S from equation (7) one gets:

$$C_W (\$) = 4.92IWp / (ab + ah + bh) \quad (17)$$

where p = cost of waste disposal (\$/GBq) for the activated ^{59}Fe .
Therefore, the total cost for radiation protection X is calculated as:

$$\begin{aligned} X &= (C_s + C_e + C_w) (\$) \\ &= (a+x).(b+x) - ab)hs + (a+x).(b+x)l + 4.92IWp / (ab + ah + bh) \end{aligned} \quad (18)$$

ESTIMATION OF THE COST OF RADIOLOGICAL HEALTH DETRIMENT

The cost of radiological health detriment can be expressed mathematically as follows:

$$Y = \beta \Lambda(x) \quad (19)$$

where

β = cost of unit collective dose for radiation protection (\$/person.Sv).

$\Lambda(x)$ = collective dose equivalent (μSv) for the shield thickness.

By substituting the value of H_t from equation (2), the collective dose equivalent $\Lambda(x)$ could be expressed as follows:

$$\Lambda(x) = 2.4\rho\eta\tau NT \exp\left\{(-x/\lambda)/(0.5a+x)^2\right\} \quad (20)$$

where ρ = ratio of the average permissible dose equivalent H_L and maximum dose equivalent H_b , η = occupancy factor, τ = cyclotron operation factor = 8760 h/y, N = number of exposed person, T = expected life of the shielding installation (y)

I = proton current at target (μA).

By substituting the value of $\Lambda(x)$ from equation (20) in equation (19) the total monetary value of the radiological health detriment is expressed as:

$$Y = 2.1\beta\rho\eta\tau NT \exp\left\{(-x/\lambda)/(0.5a+x)^2\right\} \times 10^4 \quad (21)$$

OPTIMIZATION MODEL

Now, our fuzzy optimization model can be formulated as follows:

$$\begin{aligned} C(\tilde{x}) &= \min ((a + \tilde{x}).(b + \tilde{x}) - ab)hs + (a + \tilde{x}).(b + \tilde{x})l + 4.92IWp / (ab + ah + bh) \\ &\quad + 2.1\beta\rho\eta\tau NT \exp\left\{(-\tilde{x}/\lambda)/(0.5a + \tilde{x})^2\right\} 10^4 \end{aligned} \quad (22)$$

subject to

$$H_e(t) \leq H_L \quad e = 1, 2, \dots, n \quad (23)$$

where \tilde{x} represent fuzzy parameter in the objective function with their membership function $\mu_{\tilde{x}}$. For a certain degree α together with the concept of α -cut of the fuzzy number \tilde{x} , therefore problem (22)-(23) can be understood as the following nonfuzzy problem:

$$C(x) = \min ((a+x).(b+x) - ab)hs + (a+x).(b+x)l + 4.92IWp/(ab+ah+bh) + 2.1\beta\rho\eta\tau NT \exp\{(-x/\lambda)/(0.5 a+x)^2\}10^4 \quad (24)$$

subject to

$$H_e(t) \leq H_L \quad e = 1, 2, \dots, n \quad (25)$$

$$x \in A_\alpha(\tilde{x}) \quad (26)$$

Definition 3. A feasible solution x^* is said to be α -optimal of the problem (24)-(26), if and only if there does not exist another $x \in A_\alpha(\tilde{x})$ such that $C(x) \leq C(x^*)$.

The corresponding values of parameters x^* are called α -optimal parameters.

Consider the following problem with single objective function

$$C(x) = \min ((a+x).(b+x) - ab)hs + (a+x).(b+x)l + 4.92IWp/(ab+ah+bh) + 2.1\beta\rho\eta\tau NT \exp\{(-x/\lambda)/(0.5 a+x)^2\}10^4 \quad (27)$$

subject to

$$H_e(t) \leq H_L \quad e = 1, 2, \dots, n \quad (28)$$

$$g \leq x \leq G \quad (29)$$

where g and G are lower and upper bounds on x respectively resulting from the relation $x \in A_\alpha(x)$.

Table 1. shows the category and values of the monetary cost, physical and operational parameters.

Table 1. Summarizes the values of the input parameters used in the optimization process.

Item	Description	Value	Category
β	Cost of radiation protection (\$/person.Sv)	400,000	Cost parameters
S	Cost of shielding concrete (\$m ⁻³)	300	
L	Cost of real estate, surface area (\$m ⁻²)	1000	
P	Cost of radioactive waste disposal (\$GBq ⁻¹)	100	
λ	Neutron attenuation length of concrete (m)	0.126	Physical parameters
T	Expected life of the shielding (y)	20-50	
a	Length of the vault (m)	5-10	
b	Width of the vault (m)	5-10	
h	Height of the vault (m)	5-10	
W	Weight of iron parts (Kg)	50-200	Operational parameters
N	Number of exposed person	10-50	
η	Occupancy Factor	0.2-1.0	
I	Proton beam current (μ A)	100-400	
ρ	Allowable dose equivalent	0.2-1.0	

SOLUTION ALGORITHM

Now we summarize an algorithm steps to determine the cost of radiation shielding and its thickness. The calculation steps can be described as follows:

- Step1. Let B be a set of some selected points around the cyclotron building.
- Step2. Select a point from the set B . For this point, specify a group of monetary cost (β, s, l, p) , physical parameters (λ, T, a, b, h, W) and operational parameters (N, η, I, ρ) . Remove this point from set B .
- Step 3. Set a certain degree $\alpha = \alpha^* \in [0, 1]$. Determine the points (a, m, b) for fuzzy number \tilde{x} in view of the values of x which are addressed in reference [1].
- Step 4. Specify the dose equivalent rate at this point and the cost of radiation protection $X > 0$.
- Step 5. Set optimization conditions (equations (3) and (27)). Minimum C , minimum X and minimum Y .
- Step 6. Set values for a group of monetary cost, physical parameters and operational parameters which are already specified in step 2.
- Step 7. Solve the problem by using the nonlinear form of Win QSB software.
- Step 8. Ask the decision maker about the obtained solution, if he (she) is satisfied with the calculated thickness and its cost of shielding, then go to step 9, else go to step 3.
- Step 9. If set B is empty then stop, else go to step 2.

The optimization calculation was executed using Win QSB software. A personal computer with 2.6 GHz Pentium processor and 256 MB RAM was used. For different values of $\alpha \in [0,1]$, the important results are shown in tables 2 and 3 for a proton beam current 400 μA .

Table 2. Summary of the results of optimization shielding calculations for the genetic approach.

Point	Group 1				Group 2			
1	$N = 20, T = 25 \text{ y}, W=100 \text{ kg},$ $H_L = 1.0 \text{ mSv/y}, \eta = 0.2$				$N = 20, T = 25 \text{ y}, W = 100 \text{ kg},$ $H_L = 20 \text{ mSv/y}, \eta = 1.0$			
	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t
	0.2	2.89	3.803 E+05	3.8 E-03	0.6	2.97	3.835 E+5	1.7 E-03
	0.4	2.96	3.823 E+05	2.1 E-03	0.8	2.98	3.841 E+5	1.6 E-03
2	Group 3				Group 4			
	$N = 15, T = 25 \text{ y}, W=120 \text{ kg},$ $H_L = 0.5 \text{ mSv/y}, \eta = 0.4$				$N = 10, T = 25 \text{ y}, W = 200 \text{ kg},$ $H_L = 0.8 \text{ mSv/y}, \eta = 0.7$			
	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t
	0.6	2.98	3.850 E+05	1.5 E-03	0.9	3.1	3.852 E+5	1.3 E-03
	0.8	2.99	3.851 E+05	1.4 E-03	1.0	3.18	3.853 E+5	1.2 E-03

Table 3. Summary of the results of optimization shielding calculations for our approach.

Point	Group 1					Group 2				
1	$N = 20, T = 25 \text{ y}, W=100 \text{ kg},$ $H_L = 0.2 \text{ mSv/y}, \eta = 0.2$					$N = 15, T = 25 \text{ y}, W = 150 \text{ kg},$ $H_L = 0.4 \text{ mSv/y}, \eta = 0.3$				
	α	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t	α	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t
	0.3	0.2	2.85	3.798 E+05	3.8 E-03	0.5	0.6	2.97	3.835 E+5	1.7 E-3
	0.2	0.4	2.9	3.815 E+05	2.1 E-03	0.6	0.8	2.98	3.841 E+5	1.6 E-3
2	Group 3					Group 4				
	$N = 15, T = 25 \text{ y}, W=120 \text{ kg},$ $H_L = 0.5 \text{ mSv/y}, \eta = 0.4$					$N = 10, T = 25 \text{ y}, W=200 \text{ kg},$ $H_L = 0.8 \text{ mSv/y}, \eta = 0.7$				
	α	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t	α	ρ	$x \text{ (m)}$	$X(i,k) \text{ \$}$	H_t
	0.4	0.6	2.99	3.848 E+05	1.5 E-03	0.8	0.9	3.1	3.850 E+5	1.3 E-3
	0.7	0.8	3.05	3.850 E+05	1.4 E-03	0.8	1.0	3.18	3.851 E+5	1.2 E-3

From tables 2 and 3, it is clear that our fuzzy approach gives better results than the genetic approach of Mukherjee [1]. As we know the genetic approach do not seek the optimal solution but a solution called “near optimal” solution which is almost not coincide with the optimal solution in most cases.

CONCLUSION

In this paper we have shown that an optimization approach is an appropriate tool for the radiological shielding design of concrete vaults housing cyclotron targets to produce medical radioisotopes. Also we have shown that the fuzziness is quite suitable for such practical problem, where there are different treatment plans for each value of α . We proposed a solution algorithm to solve the problem of concern. The merit of this approach as single optimization is that the problem is simplified and the representation is more realistic and practical. This is because we are treating a fuzzy and not well defined problem as it is.

In future research, an optimization approach is needed to determine type of materials which will be used to construct the shielding of medical cyclotron.

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