NUMERICAL CALCULATIONS OF THE ELECTROMAGNETIC RADIATION IN TWO DIFFERENT DIELECTRICS

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The time dependent Maxwell's Equations in closed domains filled with heterogeneous dielectrics are solved. A conducting metal wire carrying alternating current may be imbedded in them. The electromagnetic radiation of the wire is considered, through the dielectrics. Results obtained are compared with similar published cases.

1. INTRODUCTION

The study of the electromagnetic wave in bounded or unbounded media is very important nowadays. Recent engineering advances have resulted in ultra-wide band electromagnetic sources. These sources find applications in radar devices, ground penetrating imaging system, non-destructive evaluation of concrete structures, electronic on-chip interconnects and the widespread modern communication system.

The propagation of the plane EMW, in homogeneous linear isotropic dielectrics, has infinite extent. We shall investigate the effect of a discontinuity in the medium of propagation. The perfect dielectric is one with zero conductivity. So, there is no loss or absorption of power in propagation through the dielectric.

2. FORMULATION OF THE EQUATIONS

The time varying current $\overline{J}(\overline{r},t)$ gives rise to the time varying electric and magnetic fields $\overline{E}(\overline{r},t)$ and $\overline{H}(\overline{r},t)$ that satisfy Maxwell's equations. In a linear isotropic dielectric medium, these equations are given by:

$$(2.1) \qquad \overline{\nabla}\times\overline{E}(\overline{r},t) = -\mu\frac{\partial}{\partial t}\overline{H}(\overline{r},t) \qquad (Faraday's\ Law)$$

$$(2.2) \qquad \overline{\nabla}\times\overline{H}(\overline{r},t) = \varepsilon\frac{\partial}{\partial t}\overline{E}(\overline{r},t)+\overline{J}(\overline{r},t) \qquad (Ampere's\ Law)$$

$$(2.3) \qquad \overline{\nabla}\cdot\overline{E}(\overline{r},t) = \frac{1}{\varepsilon}\rho_v(\overline{r},t)$$

$$(2.4) \qquad \overline{\nabla}\cdot\overline{H}(\overline{r},t) = 0$$

ε & μ are constants (Permittivity & Permeability of a medium). If ω is the angular radian frequency of the sinusoidally time varying electric and magnetic fields $\overline{E}$ & $\overline{H}$, equations (2.1) and (2.2) are in the phasor notations are given [1, 2] as:

$$(2.5) \qquad \overline{\nabla} \times \overline{\overline{E}} = -i\omega\mu_1 \overline{\overline{H}}$$

$$(2.6) \qquad \overline{\nabla} \times \overline{\overline{H}} = i\omega\varepsilon_1 \overline{\overline{E}} + \sigma_1 \overline{\overline{E}} = i\omega\varepsilon'_1 \overline{\overline{E}}$$

where $\varepsilon'_1 = \varepsilon_1 + \frac{\sigma_1}{i\omega}$ is known as the effective permittivity of the dielectric material. $\varepsilon'_1, \varepsilon_1, \mu_1$ *and* σ_1 are respectively the effective permittivity , permittivity, perm-eability and conductivity of the dielectric material in which the guided waves propagate in medium (1). There are similar equations for medium (2).

We shall consider the transverse magnetic (T M), set of equations where the electric field is a scalar while the magnetic field is a two dimensional vector, in this case the electric field intensity-vector $\overline{\overline{E}}$ can have $\overline{E}_z$ component. The z-direction is assumed to be the direction of wave propagation. So, the wave equation satisfied will be:

$$\frac{\partial^2 \overline{E}_z}{\partial x^2} + \frac{\partial^2 \overline{E}_z}{\partial y^2} = -h^2 \overline{E}_z$$

Let $\tau = ct = \frac{t}{\sqrt{\mu\varepsilon}}$ *and* $z = \sqrt{\frac{\mu}{\varepsilon}}$ which is the impedance, and c is the speed of light. The TM equations are

$$(2.7) \qquad \frac{\partial \overline{E}_z}{\partial \tau} = z\left(\frac{\partial \overline{H}_y}{\partial x} - \frac{\partial \overline{H}_x}{\partial y}\right)$$

or, it can be written as

$$(2.8) \qquad \frac{\partial \overline{H}_y}{\partial x} - \frac{\partial \overline{H}_x}{\partial y} = i\omega\varepsilon'_1 \overline{E}_z$$

Also, we can put $H_z = 0$ in TM condition for all values of x and y.

$$(2.9) \qquad \frac{\partial^2 \overline{H}_y}{\partial x^2} = -h^2 \overline{H}_y$$

where $h^2 = \gamma^2 + \omega^2 \mu_1 \varepsilon_1, \quad \gamma = \alpha + i\beta$. (2.9) is the wave vector equation.
We can write:

$$(2.10) \qquad \frac{\partial \overline{H}_x}{\partial \tau} = -\frac{1}{z}\frac{\partial \overline{E}_z}{\partial y}$$

$$(2.11) \qquad \frac{\partial \overline{H}_y}{\partial \tau} = \frac{1}{z}\frac{\partial \overline{E}_z}{\partial x}$$

$$(2.12) \qquad \frac{\partial \overline{E}_z}{\partial \tau} = z\left(\frac{\partial \overline{H}_y}{\partial x} - \frac{\partial \overline{H}_x}{\partial y}\right)$$

where γ is the complex propagation constant, α and β are real constants.

3. APPLICATION TO CERTAIN PROBLEMS

The first problem we consider two adjacent lossless media with constants $\varepsilon_1, \varepsilon_2, \mu_1$ & μ_2. The boundary conditions are:

$$(3.1) \qquad \overline{E}_y(x=0,z)=0$$

$$(3.1) \qquad \overline{E}_y(x=a,z)=0$$

The general solution of Eq. (2.9) is:

$$(3.3) \qquad \overline{H}y=\left[C_1\sin(hx)+C_2\cos(hx)\right]e^{-\gamma z}$$

From the equations

$$\overline{H}_y=\overline{H}_{y_0}\,e^{-\gamma z}$$

$$\overline{H}_x=\overline{H}_{x_0}\,e^{-\gamma z}$$

Substitute Eq. (3.3) into the equations:

$$(3.4) \qquad \frac{\partial\overline{H}_y}{\partial x}=i\omega\varepsilon_1\overline{E}_z$$

First we determine $\overline{E}_z$ by substitution of Eq. (3.3) into Eq. (3.4)

$$(3.5) \qquad \overline{E}_z(x,z)=\frac{h}{i\omega\varepsilon_1}\left[C_1\cos(hx)-C_2\sin(hx)\right]e^{-\gamma z}$$

According to the boundary conditions we have:

$$(3.6) \qquad \overline{E}_z(x,z)=\frac{ih}{\omega\varepsilon_1}C_2\sin(hx)\,e^{-\gamma z}$$

From Eqs. (2.1) and (2.2) expressing $\overline{\nabla}\times\overline{\overline{E}}$ *and* $\overline{\nabla}\times\overline{\overline{H}}$ in terms of their rectangular coordinates. We obtain

$$\overline{\nabla}^2\overline{\overline{E}}=k^2\,\overline{E}$$

$$\overline{\nabla}^2\overline{\overline{H}}=k^2\,\overline{H}$$

where $k^2=-\omega^2\mu_1\varepsilon'_1=i\omega\mu_1\left(\sigma_1+i\omega\varepsilon_1\right)$.

For isotropic, lossless medium [3] characterized by the real constant parameters ε & μ, the uniform plane wave solution is in the form:

$$(3.7) \qquad \overline{E}=\overline{E}_o e^{i\overline{k}\cdot\overline{r}}$$

$$(3.8) \qquad \overline{H}=\overline{H}_o e^{i\overline{k}\cdot\overline{r}}$$

These two equations satisfy Maxwell's equations. $\overline{k}$ is related to the frequency ω by: $\overline{k}=\frac{\omega}{c}\sqrt{\mu\varepsilon}$

$$(3.9) \qquad \overline{H}_0=\frac{1}{\eta}\left(\overline{k}\times\overline{E}_0\right)$$

$$(3.10) \qquad \overline{E}_0=-\eta\left(\overline{k}\times\overline{H}_0\right)$$

$\eta=\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu}{\varepsilon}}$, μ is the intrinsic wave impedance, both k & μ are real. The planes of constant phase are normal to $\overline{k}$ and travel with the phase velocity: $v_p=\frac{\omega}{k}\overline{k}=\frac{c}{\sqrt{\mu\omega}}\overline{k}$. Eq. (3.9) and (3.10) are written as:

$$\overline{H} = \frac{1}{\eta}\left(\overline{k} \times \overline{E}\right)$$

$$\overline{E} = -\eta\left(\overline{k} \times \overline{H}\right)$$

So,

$$\overline{k} \times \overline{E}_0 = \omega\mu_0\mu\overline{H}_0 \qquad (3.11)$$

$$\overline{k} \times \overline{H}_0 = -\omega\varepsilon_0\varepsilon\overline{E}_0 \qquad (3.12)$$

Also, Eq. (2.9) has the solution [1] then

$$\frac{\partial^2 E}{\partial z^2} = h^2 E \qquad \& \qquad \frac{\partial^2 H}{\partial z^2} = -h^2 H$$

$$E_x = E_0\, e^{-hz} \qquad \& \qquad H_y = H_0\, e^{hz}$$

The phase shift β is given as:

$$\beta = \frac{2\pi}{\lambda} \qquad \& \qquad v_p = f\lambda = \frac{\omega}{\beta}$$

4. REFLECTION BY A PERFECT DIELECTRIC

When a plane wave is incident normally on the surface of a perfect dielectric, part of energy is transmitted and part of it is reflected. Consider the case of plane wave traveling in the x-direction incident on the boundary x=0 plane, $\overline{E}_i$ is the incident EF striking the boundary, $\overline{E}_r$ is the reflected EF leaving the boundary in the first medium, and $\overline{E}_t$ is the EF propagated in the second medium.

Also $\overline{H}_i$, $\overline{H}_r$ & $\overline{H}_t$. Therefore

$$\overline{E}_i = \eta_1\overline{H}_i$$

$$\overline{E}_r = -\eta_1\overline{H}_r$$

$$\overline{E}_t = \eta_2\overline{H}_t$$

The continuity of tangential component of $\overline{E}$ & $\overline{H}$ are

$$\overline{H}_i + \overline{H}_r = \overline{H}_t$$

$$\overline{E}_i + \overline{E}_r = \overline{E}_t$$

$$\overline{H}_i + \overline{H}_r = \frac{1}{\eta_1}\left(\overline{E}_i - \overline{E}_r\right) = \overline{H}_t = \frac{1}{\eta_2}\left(\overline{E}_i + \overline{E}_r\right)$$

Then

$$\eta_2\left(\overline{E}_i - \overline{E}_r\right) = \eta_1\left(\overline{E}_i + \overline{E}_r\right)$$

$$\frac{\overline{E}_r}{\overline{E}_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\frac{\overline{E}_t}{\overline{E}_i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Also,

$$\frac{\overline{H}_r}{\overline{H}_i} = -\frac{\overline{E}_r}{\overline{E}_i} = \frac{\eta_1 - \eta_2}{\eta_2 + \eta_1}$$

$$\frac{\overline{H}_t}{\overline{H}_i} = \frac{\eta_1 \overline{E}_t}{\eta_2 \overline{E}_i} = \frac{2\eta_1}{\eta_2 + \eta_1}$$

5. UNIFORM PLANE WAVE IN THE PRESENCE OF LOSSY MEDIUM

The presence of conductivity in a medium modifies some what the nature of the EMW propagation. In a lossy medium the EF produces the conduction current. This current gives rise to the Jule heat loss. This loss decreases the energy carried by the wave. So, the amplitude of the decrees (damped wave) as it travels through the conducting medium. Maxwell's equations in these cases take the form:

$$\overline{D} = \varepsilon_0 \varepsilon \overline{E},$$
$$\overline{B} = \mu_0 \mu \overline{H},$$
$$\overline{J}_c = \sigma \overline{E}$$

Therefore

$$\overline{\nabla} \times \overline{E} = i\omega\mu_0 \mu \overline{H},$$
$$\overline{\nabla} \times \overline{H} = -i\omega\varepsilon_0 \varepsilon_c \overline{E},$$
$$\varepsilon_c = \varepsilon + i\varepsilon_d$$

ε_d is the complex dielectric constant and is given as:

$$\varepsilon_d = \frac{\sigma}{\omega\varepsilon_0 \varepsilon}$$

From equations (3.11) & (3.12) we obtain:

$$\overline{K} \times \overline{E}_0 = \omega\mu_0 \mu \overline{H}_0$$
$$\overline{K} \times \overline{H}_0 = -\omega\varepsilon_0 \varepsilon \overline{E}_0$$
$$\overline{K} \cdot \overline{H}_0 = 0$$
$$\overline{K} \times \overline{E}_0 = 0$$
$$\overline{K}^2 = \overline{K}_0 \mu \varepsilon_c$$

The phase vector $\overline{K}_1$ and the attenuation vector $\overline{K}_2$, for the complex wave vector $\overline{K}$ is:

$$\overline{K} = \overline{K}_1 + i\overline{K}_2$$
$$\overline{K}_1^2 - \overline{K}_2^2 = \overline{K}_0^2 \mu\varepsilon$$
$$\overline{K}_1 \cdot \overline{K}_2 = K_1 K_2 \cos\zeta = \frac{1}{2}\overline{K}_0^2 \mu\varepsilon\varepsilon_d \qquad (5.1)$$

where ζ is the space angular between $\overline{K}_1$ & $\overline{K}_2$.

It is clear that, when ω & σ are non-zero. So, $\overline{K}_1$ & $\overline{K}_2$ are also non-zero> Also, we can say that, if there is loss, the fields must suffer attenuation.

For non-negative character of μ & σ, Eq. (5.1) has the condition:

$$0 < \cos\zeta \le 1, \qquad if \quad \omega > 0$$

$$or\ 0 \le \zeta < \frac{\pi}{2}, \qquad if \quad \omega > 0$$

To determine the phase constant K_1 and the attenuation constant K_2, we solve

$$K_1^2 - K_2^2 = K_0^2 \mu\varepsilon$$

$$K_1 K_2 = \frac{1}{2} K_0^2 \mu\varepsilon\varepsilon_d$$

Then we have

$$(5.2) \qquad K_1 = K_0 \sqrt{\mu\varepsilon}\left[\frac{1}{2}\sqrt{1+\varepsilon_d^2}+1\right]^2 \ge K_0\sqrt{\mu\varepsilon}$$

$$(5.3) \qquad K_2 = K_0 \sqrt{\mu\varepsilon}\left[\frac{1}{2}\sqrt{1+\varepsilon_d^2}-1\right]^2$$

So, we can say that:

$$(5.4) \qquad K_c = K_1 + iK_2$$

and depends on $K_0\sqrt{\mu\varepsilon}$ which is the place constant of uniform wave in a lossless medium, and also depends on ε_d, the loss tangent.

If ε_d <<1 a medium has a small loss> This means that the displacement current at the same frequency is large compared to the conduction current.

The Binomial expansions of equations (5.2-5.4) is

$$K_1 \approx K_0\sqrt{\mu\varepsilon}\left(1+\frac{1}{8}\varepsilon_d^2+\ldots\right) \approx K_0\sqrt{\mu\varepsilon}$$

$$K_2 \approx K_0\varepsilon_d\sqrt{\mu\varepsilon}\left(1-\frac{1}{8}\varepsilon_d^2+\ldots\right) \approx \sigma\sqrt{\frac{\mu_0\mu}{4\varepsilon_0\varepsilon}}$$

$$K_c \approx \sqrt{\frac{\mu_0\mu}{\varepsilon_0\varepsilon}}\left(-\frac{3}{8}\varepsilon_d^2-\frac{1}{2}i\,\varepsilon_d+\ldots\right) \approx \sqrt{\frac{\mu_0\mu}{4\varepsilon_0\varepsilon}}$$

When ε_d >>1 as the medium has layer loss in good conductors. In which the conduction current exceeds greatly the displacements current. In this case, the binomial expansions of equations (5.2-5.4) is

$$K_1 \approx K_0\sqrt{\frac{\mu\varepsilon\varepsilon_d}{2}}\left(1+\frac{1}{2}\varepsilon_d^{-1}+\ldots\right) \approx \sqrt{\frac{\omega\mu\mu_d\sigma}{2}}$$

$$K_2 \approx K_0\sqrt{\frac{\mu\varepsilon\varepsilon_d}{2}}\left(1-\frac{1}{2}\varepsilon_d^{-1}+\ldots\right) \approx \sqrt{\frac{\omega\mu\mu_d\sigma}{2}}$$

$$K_c \approx \sqrt{\frac{\omega\mu_0\mu}{2\sigma}}(1-i)$$

In non-dispersive media $K_1,\ K_2\ and\ K_c$ vary directly as $\sqrt{\omega}$ and the skin depth is:

$$\delta \approx \sqrt{\frac{2}{\omega\mu_0\mu\sigma}}$$

which is the distance that a plane wave is normally considered to penetrate the medium. δ is the skin depth of a wave in a conducting medium.

6. NUMERICAL APPROACH

The methods applied to equations (2.10) to (2.12) have two approaches the first is the exact solution [4]. The results obtained are given in figures 1-2 for an open domain. Figures 3-4 presents the data for two dielectrics (air and lossless media).
The second approach is using an explicit finite difference method for solving the same 3 equations. Figures 5 and 6 represent surface and contour of E_z at time 10. While Figure 7 plots logarithm of $\|error\|_2$ as a function of time.

We can conclude that the study is very important to investigate the environment in which we live, as the electromagnetic radiation is applied in many widespread new technologies.

REFRENCES

[1] E. Jordan and E. Balmain, Electro Magnetic Wave of Radial Systems, 1982.
[2] W. Miah, Fundamentals of Electro Magnetic, 1982.
[3] C.Chen, Theory of Electro Magnetic Wave a Coordinate-Free Approach,1983.
[4] A.Yeft and P. Petrpoulos, A non-dissipative Staggered Fourth-order Accurate Explicit Finite Difference Scheme for the Time-Domain Maxwell's Equations, NASA/CR-1999-209514.

Results for open domain:

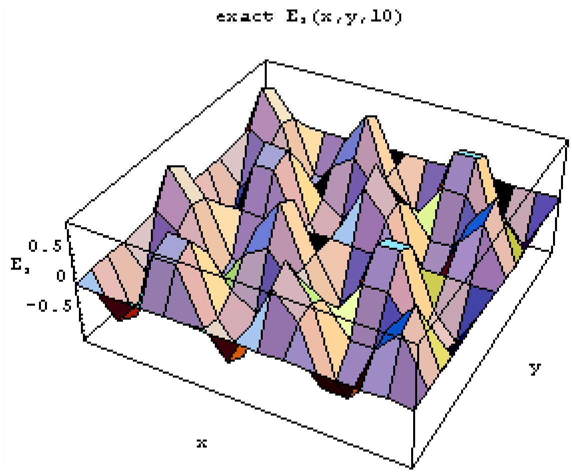

Figure 1. Surface of the exact solution $E_z(x, y, t) = \sin 3\pi x \sin 4\pi y \cos 5\pi t$ at time 10.

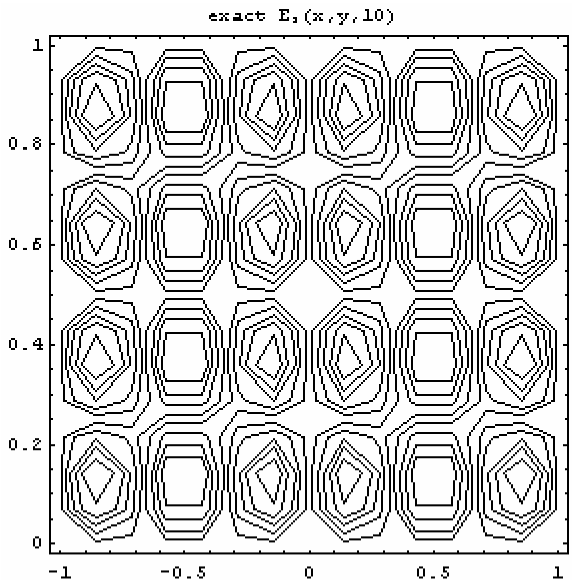

Figure 2. Contour of the exact solution $E_z(x,y,t) = \sin 3\pi x \, \sin 4\pi y \, \cos 5\pi t$ at time 10.

Results for two media:

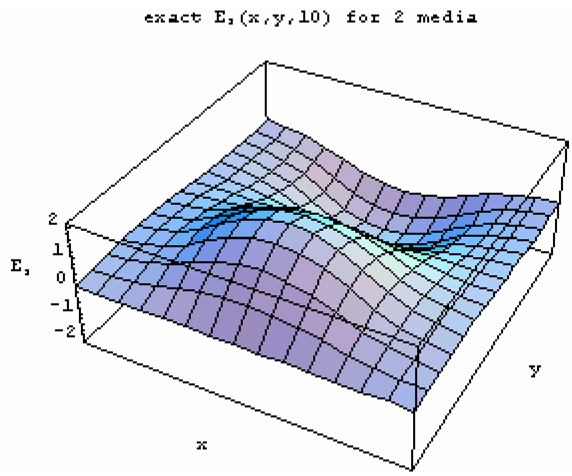

Figure 3. Surface of the exact solution $E_z(x,y,t)$ for two media at time 10.

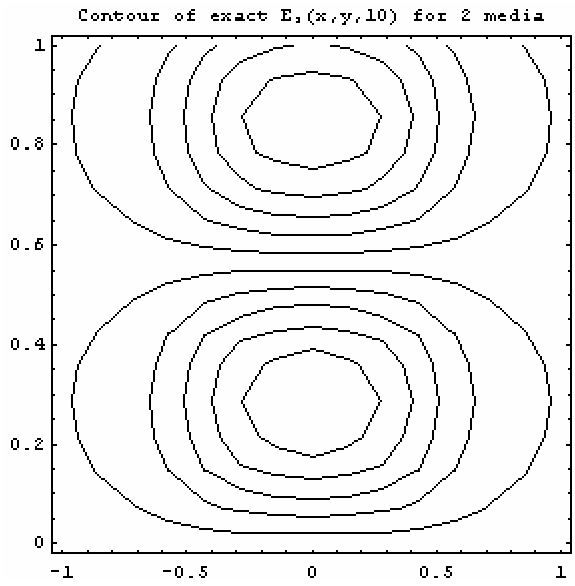

Figure 4. Contour of the exact solution $E_z(x, y, t)$ for two media at time 10.

Where

$$E_z(x,y,t) = \begin{cases} 2\cos\frac{2\pi}{3}x \ \cos\omega t \ \sin k_y y, & |x| \le \frac{1}{2} \\ e^{\frac{\pi}{\sqrt{3}}} \ e^{\frac{-2\pi}{\sqrt{3}}|x|} \cos\omega t \ \sin k_y y, & |x| \ge \frac{1}{2} \end{cases}$$

and $\omega = 4\pi$ *and* $k_y = \frac{2\pi}{3}\sqrt{7}$.

Results for perfect walls conductors:

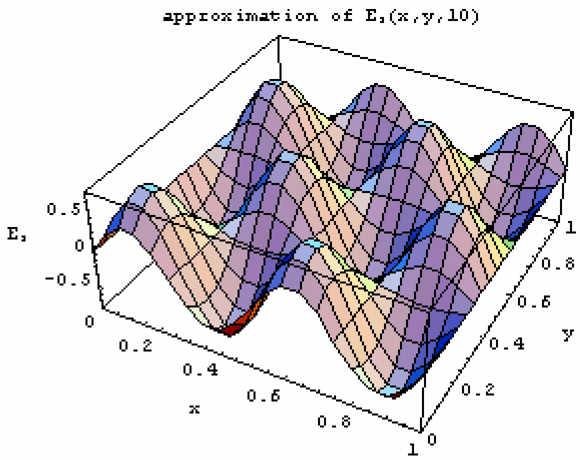

Figure 5. Surface of the approximate solution $E_z(x, y, t)$ for perfect conductor walls time 10.

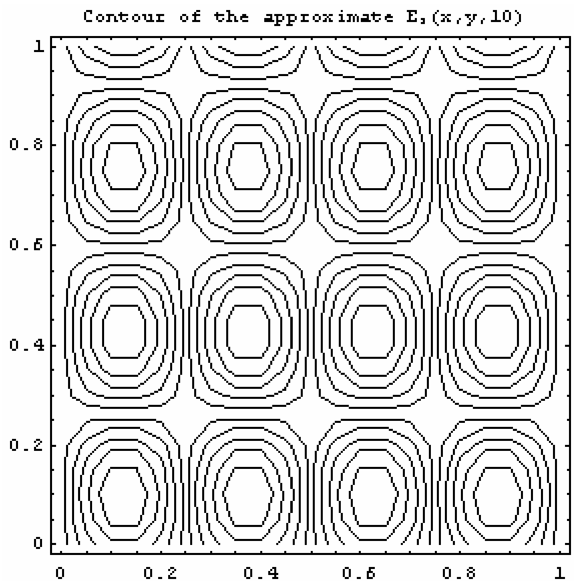

Figure 6. Contour of the approximate solution $E_z(x, y, t)$ for perfect conductor walls at time 10.

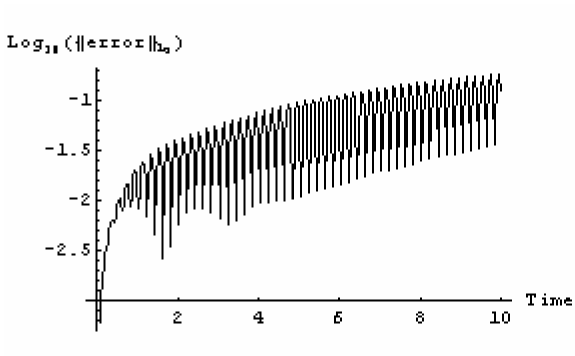

Figure 7. Plot of logarithm of $\|error\|_2$ as a function of time.

Where the initial and boundary conditions [4], are given as:

$$E_z(x, y, 0) = \sin 3\pi x \sin 4\pi y$$
$$H_x(x, y, \tfrac{\Delta t}{2}) = -\tfrac{4}{5}\cos(3\pi x - \tfrac{5\pi\Delta t}{2})\sin 4\pi y$$
$$H_y(x, y, \tfrac{\Delta t}{2}) = -\tfrac{3}{5}\sin(3\pi x - \tfrac{5\pi\Delta t}{2})\sin 4\pi y$$
$$E_z(0, y, t) = -\sin 5\pi t \sin 4\pi y$$
$$E_z(1, y, t) = \sin(3\pi x - 5\pi t)\sin 4\pi y$$
$$E_z(x, 0, t) = 0, \quad E_z(x, 1, t) = 0$$